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AN ANALYSIS OF METHODS FOR EXTRACTING AERODYNAMIC COEFFICIENTS FROM TEST DATA

UNIVERSITY OF FLORIDA

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EGLIN AIR FORCE BASE, FLORIDA

An Analysis Of Methods For Extracting
Aerodynamic Coefficients From Test Data

Donald C. Daniel

FOREWORD

This analysis was conducted by the University of Florida, Gainesville, Florida, under Contract F08635-71-C-0080 with the Air Force Armament Laboratory, Eglin Air Force Base, Florida. The program monitor for the Armament Laboratory was Dr. George B. Findley (DLGC). This effort was conducted during the period January 1971 to February 1973 and was partially supported by the Air Force Office of Scientific Research (AFOSR) under its project 9871.

The principal investigators for the University of Florida were Drs. T. E. Bullock and M. H. Clarkson.

This report is based on a dissertation previously submitted by the author to the Graduate School of the University of Florida in partial fulfillment of the requirements for the degree Doctor of Philosophy.

This technical report has been reviewed and is approved.

RANDALL L. FETTY, Colonel, USAF
Chief, Product Assurance Division

ABSTRACT

An analysis of numerical methods for extracting aerodynamic coefficients from dynamic test data has been conducted. The emphasis of the analysis is on the effects that random measurement errors in the data and random disturbances in the system have on the accuracy with which the coefficients for linear and nonlinear systems can be determined. Both deterministic and stochastic methods for extracting the coefficients and determining their uncertainties are considered.

The deterministic technique considered, due to Chapman and Kirk, provides excellent estimates of both linear and nonlinear static pitching moment coefficients for the range of measurement errors and system noise considered. Somewhat less accurate estimates of linear damping coefficients are obtained. Nonlinear pitch damping coefficients extracted using this deterministic technique are affected considerably by both measurement errors and system noise. The estimated standard deviations of the extracted coefficients obtained using standard techniques are generally adequate when the data being analyzed contain only measurement errors.

The stochastic approach considered demonstrates the feasibility of using an extended Kalman filter, with a parameter augmented state vector, for determining the values of the aerodynamic coefficients and their uncertainties from noisy dynamic test data.

The specific filter used generally reaches near steady-state conditions in its estimates of the parameters in less than one second. Variations in the initial parameter variances or in the estimates of the noise statistics essentially affect only the determination of the nonlinear damping parameter.

Parameter estimates obtained from the extended filter compare favorably with previously obtained results using deterministic techniques. Estimates of the parameter uncertainties provided by the filter are generally superior to those obtained with deterministic techniques particularly when system noise has corrupted the data.

iii
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TABLE OF CONTENTS

TABLE OF CONTENTS (CONCLUDED)

LIST OF FIGURES

LIST OF FIGURES (CONTINUED)

LIST OF FIGURES (CONTINUED)

ix

x

LIST OF FIGURES (CONCLUDED)

xi

LIST OF TABLES

LIST OF TABLES (CONCLUDED)

NOMENCLATURE

Symbol	Definition
A	Reference area
d	Reference length
I	Vehicle moment of inertia about an axis through the center of gravity and normal to its pitch plane
q	Dynamic pressure, $1/2\ \rho v^2$
V	Freestream velocity
$C_{m\alpha 0}$	Static pitching moment coefficient derivative, rad^{-1}
$C_{m\alpha 2}$	Static pitching moment coefficient derivative, rad^{-3}
$C_{m\alpha 4}$	Static pitching moment coefficient derivative, rad^{-5}
C_{mq0}	Pitch damping coefficient
C_{mq2}	Pitch damping coefficient, rad^{-2}
$C_{m\delta\alpha}$	Static pitching moment coefficient at $\alpha=0$
α	Pitch angle
ρ	Freestream density
B^{-1}	The inverse of the matrix B
B^T	The transpose of the matrix B
$E[g(X)]$	Expected value of $g(X)$ $$E[g(X)] = \int_{-\infty}^{\infty} g(\xi)F_X(\xi)\ d\xi$$ where $F_X(\xi)$ is the probability density function of the random variable X
δ_{ij}	Kronecker delta
$\delta(t-t')$	Dirac delta function

SECTION I

INTRODUCTION

1. Subject Matter

This report presents an analysis of methods for determining aerodynamic coefficients of flight vehicles for dynamic test data. This determination is accomplished by finding numerical values of the aerodynamic coefficients appearing in the equations of motion such that the solutions to these equations are adequate representations of the given test data. When this determination has been made, the coefficients are said to have been extracted from the dynamic data.

The emphasis of the analysis is placed on the effect that random measurement error in the data and random disturbances in the system have on the accuracy with which the coefficients for linear and nonlinear systems can be determined. Both deterministic and stochastic system models for extracting the coefficients and determining their uncertainties are considered.

2. Historical Background

The determination of aerodynamic coefficients from dynamic data is generally agreed to have begun with the work of Fowler, Gallop, Lock, and Richmond[1] in the first quarter of this century. Their basic technique was concerned with determining moment and damping characteristics of artillery shells by firing these projectiles through spaced cards and reconstructing the pitch and yaw angle time histories by observing the obliqueness of the holes that resulted when the shells passed through the cards. Their technique of data measurement is still in use at some ballistic ranges to this date.[2]

Nielsen and Synge[3] later clarified the linear theory of Fowler et al in their work during, and immediately following, World War II.

Coefficient extraction techniques that are currently in use at many ballistic and wind tunnel facilities evolved from the work of Murphy[4,5] and Nicolaides,[6] both of whom have considered various combinations of degrees of freedom for a variety of flight vehicles. Their aerodynamic coefficient extraction techniques feature a least squares fit to dynamic data of the exact solutions of linear equations of motion, or approximate closed form solutions of slightly nonlinear equations of motion using the quasi-linearization technique of Kryloff and Bogoliuboff.[7] Ex-

tensions of the work of Murphy and Nicolaides, particularly for more complex nonlinear systems, have been made by Eikenberry and Ingram.[9]

The requirement for closed form solutions of the equations of motion has recently been eliminated by the formulation of least squares techniques which fit numerical solutions of the equations of motion to dynamic data. The minimization of the least squares criterion function involves differential corrections, as do the techniques of Murphy and Nicolaides; the required partial derivatives are determined by numerically integrating parametric differential equations which are derived from the equations of motion. Contributions to this numerical coefficient extraction technique have been made independently by Chapman and Kirk,[10] Knadler,[11] Goodman[12,13] and Meissinger,[14] although the method is usually referred to as the Chapman-Kirk technique. The computational requirements of this technique are sometimes extensive,[15] but this is usually outweighed by the fact that it can be used to analyze highly nonlinear aerodynamic forces and moments.[16]

All of the coefficient extraction techniques that have been discussed to this point are deterministic in nature in that the modeling of the equations of motion does not account for random disturbances in the system. Most angular or translational motion data obtained from ballistic ranges or wind tunnel dynamic tests have, nevertheless, been affected by random system noise and random measurement errors. The effects of noise of these types on the accuracy of coefficients extracted from dynamic data using deterministic techniques are of current interest.[17] With the exception of Ingram's[9] partial analysis of noise effects on Eikenberry's "Wobble" program,[8] however, little has been reported concerning these effects. One of the primary goals of the research reported here is to show some effects of random measurement errors and system noise on the accuracy of aerodynamic coefficients extracted from dynamic data using the most general of the various deterministic techniques, that of Chapman and Kirk.

In recent years increasing attention has been given to parameter and state variable estimation through the use of stochastic modeling of the physical system of interest. Most of this work has been done by optimal control specialists and is a direct result of pioneering contributions in linear filtering theory by Kalman[18,19] and Bucy.[19] The Kalman filter provides estimates of the states of noisy, linear dynamic systems as well as estimates of the state variable uncertainties. Bryson and Ho,[20] among others, have extended the Kalman filter theory to include estimates of the states for nonlinear systems.

2

Mehra[21] has recently proposed a maximum likelihood technique for the determination of aerodynamic coefficients from dynamic data using the Kalman filter for linear systems and the extended Kalman filter when the system is non-linear. Results verifying his proposals are, as yet, unavailable.

An additional maximum likelihood technique has been developed by Grove et al[22] and has been used with modest success by Suit.[23]

A stochastic approach to the coefficient extraction problem using an extended Kalman filter with parameter augmented state vector is developed in this report. Results obtained with this filter are also presented.

3. Scope of the Report

The remainder of this report is devoted to an analysis of the effects of random system noise and measurement errors on coefficients extracted from dynamic data using the method of Chapman and Kirk, as well as the development and evaluation of an extended Kalman filter for solving essentially the same problem. The equation of motion used in the analysis is one describing the one-degree-of-freedom pitching motion of a rigid body. Pitching moments that are both linear and nonlinear functions of angle-of-attack are considered.

The theory of the Chapman-Kirk technique is recounted briefly in Section II, followed by the analysis of noise effects on this method in Section III.

Developments leading to the formulation of the extended Kalman filter to be used are given in Section IV. Results obtained with the extended filter are presented and discussed in Section V.

A summary is given in Section VI.

Computer program listings and definitions of program variables are given in the two appendices.

THE COEFFICIENT EXTRACTION TECHNIQUE
OF CHAPMAN AND KIRK

1.   Development of the Extraction Algorithm

A brief reconstruction of the Chapman-Kirk algorithm
for extracting aerodynamic coefficients from dynamic data
is given in this section. The standard technique for
estimating uncertainties in coefficients determined from
a least squares fit of a given function to experimental
data is also presented.

The extraction technique of Chapman and Kirk has two
very basic requirements: (1) the differential equation
of motion for the body of interest must be given and
(2) a set of experimental data based on the observed motion
of the body must be available.

As an example, consider the nonlinear equation of
pitching motion for a rigid body

$$\ddot{\alpha} + (C_4 + C_6\alpha^2)\dot{\alpha} + (C_3 + C_5\alpha^2)\alpha = 0 \qquad (1)$$

subject to the initial conditions

$$\alpha(0) = C_1$$

$$\dot{\alpha}(0) = C_2$$

where $\dot{\alpha}$ indicates the derivative of α with respect to time.
Suppose, also, that the time history of the pitch angle
as recorded during some experiment, $\alpha_e(t_i)$, i=1,2,...m,
is also available.

The technique of Chapman and Kirk is used to determine
the values of the C_j (j=1,2,...,6) in Equation (1) which
result in the solution to this equation of motion being
a best fit to the test data in a least squares sense.
Thus, it is necessary to minimize the least squares cri-
terion function

$$S = \sum_{i=1}^{m} \left[\alpha_e(t_i) - \alpha_c(t_i) \right]^2 \qquad (2)$$

where $\alpha_c(t_i)$ is obtained from the solution to Equation (1).

Now it is well known that in order to determine parameters directly by a least squares fit of a given function to test data, the parameters must appear linearly in the function. This requirement is met in the problem at hand by expanding α_C in a truncated Taylor series about the numerical solution resulting from some initial estimates of the parameters of interest, C_{jo}, i.e.,

$$\alpha_c(t_i) = \alpha_{co}(t_i) + \sum_{j=1}^{6} \left(\frac{\partial \alpha}{\partial C_j} \right)_i \Delta C_j \quad , \tag{3}$$

where $\left(\dfrac{\partial \alpha}{\partial C_j} \right)$ are evaluated for $C_j = C_{jo}$ and $\Delta C_j = C_j - C_{jo}$.

Substituting (3) into (2) yields

$$S = \sum_{i=1}^{m} \left\{ \alpha_e(t_i) - \alpha_{co}(t_i) - \sum_{j=1}^{6} \left[\frac{\partial \alpha}{\partial C_j} \right]_i \Delta C_j \right\}^2 \quad . \tag{4}$$

Now, assuming that the $\left[\dfrac{\partial \alpha}{\partial C_j} \right]$ are known, Equation (4) is a

function of the ΔC_j's only. Therefore, to determine the values of these coefficients that will minimize this equation, it is necessary to take the partial derivative of Equation (4) with respect to each ΔC_j, set each of the resulting equations to zero and solve for the ΔC_j's. Carrying out these operations yields the following matrix equation

$$[A] \quad [\Delta C] = [B] \tag{5}$$

where A is, in this case, a 6 X 6 matrix with elements given by

$$A_{jk} = \sum_{i=1}^{m} \left[\frac{\partial \alpha}{\partial C_j} \right]_i \left[\frac{\partial \alpha}{\partial C_k} \right]_i \quad , \tag{6}$$

5

ΔC is a 6 X 1 column matrix, or vector, and B is a 6 X 1 column matrix with elements given by

$$B_j = \sum_{i=1}^{m} \left[\alpha_e(t_i) - \alpha_{co}(t_i) \right] \left[\frac{\partial \alpha}{\partial C_j} \right]_i \quad . \tag{7}$$

The solution to Equation (5) is

$$[\Delta C] = [A]^{-1} [B] \quad . \tag{8}$$

The solutions for the ΔC_j's obtained from Equation (8) are exactly correct only if α_c is a linear function of the C_j's as assumed by Equation (3). This condition of linearity is seldom the case, and the process must be repeated with new initial guesses,

$$C_{j1} = C_{jo} + \Delta C_{jo} \tag{9}$$

until the change in the criterion function [Equation (2)] from one iteration to the next is sufficiently small.

The algorithm just presented requires that time histories of the influence coefficients, $\frac{\partial \alpha}{\partial C_j}$, be available. These time histories are determined by numerically integrating parametric differential equations which are derived by differentiating the equation of motion with respect to each of the parameters of interest. As an example, the parametric differential equation for C_1, the initial condition of α is

$$\frac{\partial}{\partial C_1}[\ddot{\alpha} + (C_4 + C_6 \alpha^2)\dot{\alpha} + (C_3 + C_5 \alpha^2)\alpha] = 0$$

$$\frac{\partial \ddot{\alpha}}{\partial C_1} + \frac{\partial C_4}{\partial C_1} \dot{\alpha} + \frac{\partial C_6}{\partial C_1} \alpha^2 \dot{\alpha} + 2C_6 \alpha \dot{\alpha} \frac{\partial \alpha}{\partial C_1} + C_6 \alpha^2 \frac{\partial \dot{\alpha}}{\partial C_1} + \frac{\partial C_3}{\partial C_1} \alpha$$

$$+ C_3 \frac{\partial \alpha}{\partial C_1} + \frac{\partial C_5}{\partial \alpha} \alpha^3 + 3C_5 \alpha^2 \frac{\partial \alpha}{\partial C_1} = 0$$

6

Assuming that the parameters are independent of each other and that the order of differentiation can be reversed, the final form of the desired equation is

$$\frac{d^2}{dt^2}\left[\frac{\partial \alpha}{\partial C_1}\right] + C_6\alpha^2 \frac{d}{dt}\left[\frac{\partial \alpha}{\partial C_1}\right] + (C_3+3C_5\alpha^2+2C_6\alpha\dot{\alpha})\left[\frac{\partial \alpha}{\partial C_1}\right] = 0 \qquad (10)$$

subject to the initial conditions

$$\frac{\partial \alpha(0)}{\partial C_1} = \frac{\partial C_1}{\partial C_1} = 1$$

$$\frac{\partial \dot{\alpha}(0)}{\partial C_1} = \frac{\partial C_2}{\partial C_1} = 0 \qquad . \qquad (11)$$

The complete set of parametric differential equations for the given equation of motion [Equation (1)] is given in Section III.

In summary, the process for extracting numerical coefficients from test data given the system model [Equation (1)] and criterion function [Equation (2)] is

1. Estimate the numerical values of the C_j's.

2. Integrate Equation (1) to obtain $\alpha_{co}(t_i)$.

3. Determine $\left[\frac{\partial \alpha}{\partial C_j}\right]_i$.

4. Solve Equation (8) for the ΔC_j's.

5. Repeat the process with $C_{j1}=C_{jo}=\Delta C_{jo}$ until the change in Euqation (2) is sufficiently small.

7

2. Estimation of Extracted Parameter Uncertainties

The estimation of the uncertainties, or standard deviations, of parameters that have been determined by the lec _ squares fitting of a given function to test data is a well-known result available in a variety of references (see, for example, References 24 or 25).

The least square parameter uncertainties are estimated in this report by

$$\sigma_j = \sqrt{A_{jj}^{-1}} \cdot \sqrt{\frac{S}{m-K}} \tag{12}$$

where A_{jj}^{-1} is the jth diagonal element of the inverse Grammian matrix [Equations (6) and (8)], S is the sum of the squares of the residuals as given by Equation (2), m is the total number of data points, and K is the total number of parameters being determined by the fit.

SECTION III

ANALYSIS OF THE CHAPMAN-KIRK
COEFFICIENT EXTRACTION TECHNIQUE

1. <u>Coefficient Extraction Computer Program</u>

This section provides a detailed description of a one-degree-of-freedom coefficient extraction computer program based on the previously described iterative process of Chapman and Kirk. This program considers the pitching motion of a symmetric missile about a fixed point. It is used to determine values of static pitching moment coefficient derivatives, pitch damping coefficients, and a trim term so that the solution to the appropriate differential equation of motion is a best fit to test data in a least squares sense.

The program was developed to be used as an economical tool in the evaluation of the sensitivity to noise and convergency sensitivity of the Chapman-Kirk technique when operating in its least complex mode.

a. <u>Computational Equations</u>

(1) Equation of Motion

The complete equation of motion that is used in the program is

$$\ddot{\alpha} + (C_4 + C_6\alpha^2)\dot{\alpha} + (C_3 + C_5\alpha^2 + C_7\alpha^4)\alpha + C_8 = 0 \tag{13}$$

with initial conditions

$$\alpha(0) = C_1 \qquad , \qquad \dot{\alpha}(0) = C_2 \tag{14}$$

where

$$C_3 = -\frac{(C_{m\alpha o})qAd}{I} \qquad\qquad C_4 = -\frac{(C_{mqo})qAd^2}{2VI}$$

$$C_5 = - \frac{(C_{m\alpha 2}) \, qAd}{I} \qquad\qquad C_6 = - \frac{(C_{mq2}) \, qAd^2}{2VI}$$

$$C_7 = - \frac{(C_{m\alpha 4}) \, qAd}{I} \qquad\qquad C_8 = - \frac{(C_{m\delta\alpha}) \, qAd}{I}$$

(2) Parametric Differential Equations

The eight parametric differential equations for the given equation of motion [Equation (13)] and initial conditions [Equation (14)] are of the form

$$\ddot{P}_j + A\dot{P}_j + BP_j = F_j \qquad , \qquad j = 1,2,\ldots,8 \qquad\qquad (15)$$

where

$$P_j = \frac{\partial \alpha}{\partial C_j} \quad , \quad \dot{P}_j = \frac{d}{dt} \frac{\partial \alpha}{\partial C_j} \quad , \quad \ddot{P}_j = \frac{d^2}{dt^2} \frac{\partial \alpha}{\partial C_j}$$

$$A = (C_4 + C_6 \alpha^2)$$

and

$$B = (C_3 + 3C_5 \alpha^2 + 2C_6 \alpha \dot{\alpha} + 5C_7 \alpha^4).$$

The values of the initial conditions $P_j(0)$ and $\dot{P}_j(0)$ as well as the functional forms of the nonhomogeneous term F_j are given in Table I for $j=1,2,\ldots,8$.

TABLE I. INITIAL CONDITIONS AND NONHOMOGENEOUS TERMS FOR PARAMETRIC DIFFERENTIAL EQUATIONS

j	$P_j(0)$	$\dot{P}_j(0)$	F_j
1	1	0	0
2	0	1	0
3	0	0	$-\alpha$
4	0	0	$-\dot{\alpha}$
5	0	0	$-\alpha^3$
6	0	0	$-\dot{\alpha}\alpha^2$
7	0	0	$-\alpha^5$
8	0	0	-1

b. Program Description

The program is written in Fortran IV for use primarily on an IBM 360/65. The paragraphs that follow describe the functions of the main program and its four subroutines, the required imput data, and the program options. A listing of the complete program is given in Appendix I.

(1) Main Program Functions

The functions of the main program in their approximate order of occurrence are as follows:

(a) Reads and writes input data.

(b) Computes initial parameter values, [These are the C_j's that appear in Equation (13).]

(c) Calls the numerical integration subroutine ADDUM.

(d) Writes current parameter values.

(e) Computes the matrix elements required for incrementing the parameter values.

(f) Calls the matrix inversion subroutine MINV.

(g) Computes the sum of the squares of the residuals between the calculated and experimental data points.

(h) Computes the root-mean-square residual and root-mean-square error (Reference 24) of the current fit to date.

(i) Computes the estimated standard deviations (Reference 24) of the current parameter values.

(j) Tests the difference between the current and previous values of the root-mean-square error to determine if the iteration process has converged.

(k) Computes the incremental changes for the parameters (if convergence has not occurred).

(l) Computes the updated parameter values (if convergence has not occurred).

(m) Returns to (c) (if convergence has not occurred).

(n) Computes the values of the extracted coefficients from the current parameter values (after convergence is achieved).

(o) Computes the estimated standard deviations of the extracted aerodynamic coefficients (after convergence is achieved).

(p) Writes extracted aerodynamic coefficients, estimated standard deviations, and the pitch angle output that represents the final fit to the experimental data (after convergence is achieved).

(2) Subroutine Functions

The names and functions of each of the four subroutines that are used in the coefficient extraction program are given below.

ADDUM.-- Subroutine ADDUM integrates the equation of motion and parametric differential equations using a fourth order Runge-Kutta method for starting and a fourth order Adams-Bashforth predictor-corrector method for running. It calls subroutines XDOT and OUT. This subroutine is described in detail in Reference 26.

12

XDOT.-- Subroutine XDOT computes current values of first derivatives that are required by ADDUM.

OUT.-- Subroutine OUT stores the results of the numerically integrated equation of motion [Equation (13)] and parametric differential equations [Equation (15)].

MINV.-- Subroutine MINV inverts a K x K matrix using a standard Gauss-Jordan technique and is described in detail in Reference 27.

(3) Required Input Data

The program reads six categories of input data. These categories and the specific data in each are de- lineated in Appendix I. The format and units of the entries on a specific data card can be determined from the program listing and nomenclature list provided in this appendix.

(4) Program Options

This program has options for extracting various combinations of aerodynamic coefficients from the given test data, in addition to the option of extracting no coefficients and merely integrating the equation of motion. These options are controlled by the numerical value of the number of first order equations to be integrated, N, which is read by the program on the first data card. The various options are given in Table II.

TABLE II. PROGRAM OPTIONS

N	Coefficients to be Extracted
2	No coefficients are extracted. The program integrates the equation of motion for the given input data and prints the results.
8	α_o, $\dot{\alpha}_o$, $C_{m\alpha o}$
10	α_o, $\dot{\alpha}_o$, $C_{m\alpha o}$, C_{mqo}
12	α_o, $\dot{\alpha}_o$, $C_{m\alpha o}$, C_{mqo}, $C_{m\alpha 2}$
14	α_o, $\dot{\alpha}_o$, $C_{m\alpha o}$, C_{mqo}, $C_{m\alpha 2}$, C_{mq2}
16	α_o, $\dot{\alpha}_o$, $C_{m\alpha o}$, C_{mqo}, $C_{m\alpha 2}$, C_{mq2}, $C_{m\alpha 4}$
18	α_o, $\dot{\alpha}_o$, $C_{m\alpha o}$, C_{mqo}, $C_{m\alpha 2}$, C_{mq2}, $C_{m\alpha 4}$, $C_{m\delta\alpha}$

2. Analysis of the Sensitivity to
    Noise of the Chapman-Kirk Technique

The evaluation of the sensitivity to noise of the
Chapman-Kirk coefficient extraction technique has been
approached from several directions. Coefficients have
been extracted from numerous sets of computer program
generated data containing only measurement errors as well
as data containing system noise and measurement errors.

In addition to the use of artificially generated data,
some very preliminary work has been done with noisy wind
tunnel data obtained from actual experimentation. The
design of the experiment used to produce the dynamic
data is described by Turner in Reference 28 along with some
preliminary results obtained with the one-degree-of-freedom
program described earlier in this section of the report.

Since the wind tunnel experimentation is still in a
developmental stage, however, the results presented in the

14

remainder of this section are those obtained from artificially generated data.

a. Generation of Noisy Dynamic Data

The computer program UFNOISE described in detail in Reference 29 was used to generate the dynamic data from which the aerodynamic coefficients were extracted. This program simulates the pitching motion of symmetric missile oscillating in a wind stream about a fixed point, for any given initial pitch angle displacement and initial pitch angle rate, by numerically integrating the equation of motion. The program has two options for simulating system noise; it considers the magnitude and direction of the freestream velocity vector as normally distributed random variables, with programmer set mean values and standard deviations, to determine the system noise perturbation accelerations or it simply selects random perturbation accelerations which have zero mean, are normally distributed, and have a programmer set standard deviation. Both methods are essentially equivalent. New values of the system noise perturbations are randomly selected at each numerical integration step. The program also has the option of simulating random measurement errors of the pitch angle by superimposing normally distributed random noise on the output of the numerical integration.

The basic equation of motion used to generate the noisy dynamic data was a slightly simplified form of Equation (13):

$$\ddot{\alpha} + (C_4 + C_6\alpha^2)\dot{\alpha} + (C_3 + C_5\alpha^2)\alpha = w(t) \tag{16}$$

subject to

$$\alpha(0) = C_1 \quad , \quad \dot{\alpha}(0) = C_2 \quad . \tag{17}$$

The true values of the physical and aerodynamic constants used in generating the data are given in Table III. The various noise level standard deviations and mean values are discussed in the following paragraphs to which they are pertinent.

15

TABLE III. TRUE VALUES OF CONSTANTS
USED IN GENERATING DATA

Constant	Value
d(Ft)	0.333
A(Ft^2)	0.0873
I(slug ft^2)	0.1080
V_m(ft/sec)	500.0
q(lb/ft^2)	297.0
$C_{m\alpha o}$(rad-^{1})	-2.00
$C_{m\alpha_2}$(rad-^{3})	-24.5
C_{mqo}	-60.0
C_{mq_2}(rad-^{2})	-163.0
α_o(rad)	0.5235
$\dot{\alpha}_o$(rad/sec)	0.0

b. Effects of Random Measurement Errors

Aerodynamic coefficients were extracted from a total
of nine sets of data containing only measurement errors.
Each data set was made up of 201 discrete points which re-
sult from integrating the equation of motion [Equation (16)]
in increments of 0.005 second for a total of 1.000 second.

The desired mean value of the measurement errors for
each of the nine data sets was 0.0 radian. The desired
standard deviations of the random errors were 0.00146
radian, 0.00582 radian, and 0.01745 radian, with three
different sets of data being generated at each noise level.
These standard deviations are typical of measurement un-
certainty for data of this type (Reference 9).

16

The above standard deviations correspond to percent noise levels of 1.1, 4.4, and 13.2, respectively, where percent noise level is defined in Reference 30 as

$$\text{Percent Noise} = \frac{3\sigma}{\text{Approximate average peak amplitude}} \qquad (18)$$

with σ being the standard deviation of interest. The approximate average peak amplitude can be determined in a variety of ways. The value used here is the mean positive peak amplitude for the three cycles that result when the nominal equation of motion is integrated for 1.000 second.

The results of this portion of the analysis are given in Table IV and in Figures 1 through 10.

Table IV is primarily a summary of the percent error in the extracted aerodynamic coefficients together with their normalized estimated standard deviations for the various actual measurement errors, σ_m. The percent error in the extracted coefficients is defined by

$$\text{Percent Error } (C_j) = \frac{(\hat{C}_j - C_j) \cdot 100}{C_j} \qquad (19)$$

where $\hat{C}_j$ is the coefficient estimate determined with the program in the fitting of the data and C_j is the true value of the coefficient of interest. The normalized estimated standard deviations are the estimated standard deviations calculated from Equation (12) divided by the true value of the parameter of interest,

$$\text{Percent } \sigma_{jn} = \frac{\sigma_j}{C_j} \cdot 100 \qquad (20)$$

Table IV also contains the RMS residual for each fit to the noisy data and the percent noise levels based on the values of σ_m.

17

TABLE IV. SUMMARY OF RESULTS FOR EXTRACTING AERODYNAMIC COEFFICIENTS FROM DYNAMIC DATA CONTAINING RANDOM MEASUREMENT ERRORS

Data Set	σ_m (Rad)	Percent Noise	Percent Error in Extracted Coefficients and Normalized Estimated Standard Deviations				RMS Residual (Rad)
			$C_3(\sigma_3)$	$C_4(\sigma_4)$	$C_5(\sigma_5)$	$C_6(\sigma_6)$	
1	0.00158	1.2	0.07(0.39)	-0.48(0.93)	0.09(0.30)	2.24(8.38)	0.00155
2	0.00142	1.1	0.21(0.34)	0.06(0.82)	-0.41(0.26)	-1.85(7.32)	0.00136
3	0.00144	1.1	0.25(0.36)	1.34(0.86)	0.19(0.28)	-10.40(7.70)	0.00143
4	0.00587	4.4	-0.92(1.48)	0.79(3.48)	0.70(1.13)	-4.42(31.26)	0.00582
5	0.00607	4.5	2.44(1.45)	2.48(3.50)	-1.94(1.11)	-11.40(31.41)	0.00589
6	0.00560	4.3	-0.28(1.41)	0.62(3.35)	0.05(1.08)	-12.05(30.01)	0.00556
7	0.01842	14.0	-1.49(4.63)	24.94(10.89)	0.92(3.53)	-226.94(97.51)	0.01810
8	0.01672	12.5	2.90(4.02)	-2.00(9.67)	-4.18(3.00)	5.42(85.06)	0.01622
9	0.01837	14.7	0.48(4.70)	11.44(11.22)	-1.42(3.57)	-84.11(99.54)	0.01906

An example of one of the program fits to a noisy data set is shown on Figure 1. Figures 2 through 5 show the variation of the percent error in the individual extracted coefficients with measurement error. The variation of RMS residual with measurement error is shown on Figure 6. The variations of the normalized estimated standard deviations of the extracted coefficients with measurement error are shown in Figures 7 through 10.

An analysis of Table IV and Figures 2 through 10 reveals the following:

a. The extracted static pitching moment coefficient derivatives show little or no error for the entire range of measurement errors considered. (See Figures 2 and 3.)

b. The extracted pitch damping coefficients show some error for the lower noise values ($\sigma_m \leq 0.005$ radian) and deviate significantly for $\sigma_m \approx 0.018$ radian. (See Figures 4 and 5.)

c. The variation of RMS residual with measurement noise is essentially linear with a slope of unity (Figure 6). This indicates that the RMS residual, as calculated by the coefficient extraction program, is a good estimate of the amount of measurement noise in the data when this is the only type of noise present.

d. All variations of the normalized estimated standard deviations of the extracted coefficients, σ_{jn}, increase linearly with measurement error (Figures 7 through 10). Approximate values of the slopes of these variations are given in Table V. These normalized uncertainty ratios give the relative (to each other) uncertainty which can be expected when extracting coefficients from data containing measurement noise using the given coefficient extraction program.

e. The true value of a given coefficient is contained within the interval defined by its extracted value $\pm 3\sigma_j$ for every coefficient extracted from the nine sets of data.

19

TABLE V. EXTRACTED COEFFICIENT UNCERTAINTY RATIOS

Coefficient (C_j)	Approximate Normalized Uncertainty Ratio $(\Delta\sigma_j/\Delta\sigma_m)$
C_3	2
C_5	2
C_4	5
C_6	50

c. Effects of Random Measurement Errors and System Noise

For this portion of the analysis, attempts were made
to extract aerodynamic coefficients from twelve basic
sets of dynamic data. Each data set was made up of 201
discrete points resulting from integrating the equation
of motion in increments of 0.005 second for 4.00 seconds;
the pitch angle was output every four integration steps
so that the time between data points was 0.020 second.
The time between data points used in this portion of the
analysis is different from that previously used. This
change was made so that the time between data points would
correspond to that used by Turner[28] in actual experiment-
ation.

Of the twelve basic data sets, nine contain system
noise and measurement errors, whereas three contain only
measurement errors. Coefficients were extracted from these
latter three to determine if the above-mentioned change in
the time increment between data points had any appreciable
effect on the extracted coefficients or their uncertainties.
The estimates of the uncertainties were improved due pri-
marily to the fact that a longer data record was being
analyzed.

The desired mean values of the freestream velocity
vector direction fluctuations and measurement errors for
all data sets were zero. The desired standard deviation
of the measurement errors, σ_m, was 0.00582 radian
($3\sigma_m = 1.0$ degree) for all sets; the desired standard
deviation of the velocity magnitude fluctuations, σ_v,
was 5.0 ft/sec for all nine data sets containing system
noise. The desired standard deviations of the velocity
direction, σ_a, were 0.02910 radian ($3\sigma_a = 5$ degrees),
0.05820 radian ($3\sigma_a = 10$ degrees), and 0.11640 radian
($3\sigma_a = 20$ degrees), with three different sets of data
being generated at each noise level.

The random velocity fluctuations act as a forcing
function for the equation of motion and cause oscillations
even if the vehicle has no initial displacement. The
resultant maximum amplitudes of these forced oscillations
are usually within a certain magnitude or noise band width.
The widths of the noise bands for the data used in this
analysis are generally equivalent to the $3\sigma_a$ values, and
these have been used to calculate the percent system noise
values. The approximate average peak amplitude used in the
percent noise calculations is the mean positive peak ampli-
tude for the 10 cycles that result when the nominal equation
of motion is integrated for approximately four seconds.

21

The results of this portion of the analysis are given in Table VI and in Figures 11 through 19.

Table VI is a summary of the percent error in the extracted aerodynamic coefficients and their normalized estimated standard deviations along with the various noise level standard deviations, percent system noise levels, and the RMS residual of each fit to a noisy data set. The percent system noise levels are similar to those encountered experimentally by Turner in Reference 28.

TABLE VI. SUMMARY OF RESULTS FOR EXTRACTING AERODYNAMIC COEFFICIENTS FROM DYNAMIC DATA CONTAINING RANDOM MEASUREMENT ERRORS AND SYSTEM NOISE

Data Set	σ_m (Rad)	σ_v (Ft/sec)	σ_a (Rad)	Percent System Noise	Percent Error in Extracted Coefficients and Normalized Estimated Standard Deviations				RMS Residual (σ_{mr})
					$C_3(\sigma_3)$	$C_4(\sigma_4)$	$C_5(\sigma_5)$	$C_6(\sigma_6)$	
10	0.00527	0.00	0.00	0.00	-0.27(0.25)	-0.84(0.93)	0.68(0.65)	5.72(12.92)	0.00520
11	0.00589	0.00	0.00	0.00	-0.34(0.28)	0.29(1.04)	1.51(0.73)	-3.60(14.43)	0.00574
12	0.00574	0.00	0.00	0.00	0.42(0.27)	-0.36(1.01)	-0.94(0.69)	4.40(13.88)	0.00563
13	0.00611	5.05	0.02910	41.4	0.55(0.64)	8.25(2.75)	9.55(1.88)	329.59(43.58)	0.01258
14	0.00597	5.18	0.03038	43.1	4.19(0.63)	5.84(2.41)	1.35(1.63)	-44.80(34.40)	0.01244
15	0.00587	4.88	0.02828	40.2	2.87(0.55)	20.73(2.10)	-2.80(1.24)	-247.20(26.39)	0.01026
16	0.00603	5.01	0.005516	78.4	2.71(1.24)	-42.80(3.95)	6.54(3.64)	605.77(66.79)	0.02806
17	0.00562	4.90	0.06120	86.8	14.44(1.32)	-0.90(5.35)	5.44(3.87)	197.10(85.51)	0.02627
18	0.00600	4.96	0.05662	80.3	1.41(0.70)	-35.67(2.33)	14.59(2.30)	586.34(66.49)	0.01488
19	0.00572	5.08	0.11262	152.9			DIVERGED		
20	0.00547	4.85	0.11706	158.9			DIVERGED		
21	0.00571	4.96	0.12058	163.7			DIVERGED		

23

Figure 11 is an example of one of the program fits
to a noisy data set. Figures 12 through 15 show the varia-
tion of percent error for the extracted coefficients with
system noise. The variations of the normalized estimated
standard deviations of the extracted coefficients with
system noise level are shown on Figures 16 through 19.

An analysis of Table VI and Figures 12 through 19
reveals the following:

a. The errors in the linear static pitching moment
   coefficient derivative parameter, C_3, are gen-
   erally less than 5 percent for the entire range
   of system noise considered. The errors in the
   corresponding nonlinear term, C_5, are generally
   less than 10 percent. (See Figures 12 and 13.)

b. The pitch damping parameters show significant
   error for all nonzero system noise levels. The
   extracted values of the nonlinear term, C_6, are
   sometimes in error by several orders of magnitude
   at the higher noise levels. (See Figures 14
   and 15.)

c. The estimated standard deviations of the extracted
   coefficients generally increase as the system noise
   increases. The estimated standard deviations
   of coefficients extracted from dynamic data
   containing both system noise and measurement
   errors are generally too small and do not reflect
   the true uncertainty of the extracted coeffi-
   cients as was true in the previous case when
   the data contained only measurement errors.
   (See Table VI and Figures 16 through 19.)

d. All attempts at extracting the complete set of
   coefficients from data with a system noise band
   of approximately 20 degrees failed (see Table VI).
   The failures resulted when a value of C_5 was
   eventually calculated by the iterative process
   which caused the solution to the equation of
   motion to diverge.

e. The divergence problem can sometimes be circum-
   vented by not attempting to extract C_{mq2} from
   extremely noisy data. The resulting coefficients
   that are extracted, $C_{m\alpha o}$, $C_{m\alpha 2}$, and C_{mqo}, have
   accuracies comparable to those extracted from
   noisy data with a ± 10 degrees system noise band.

24
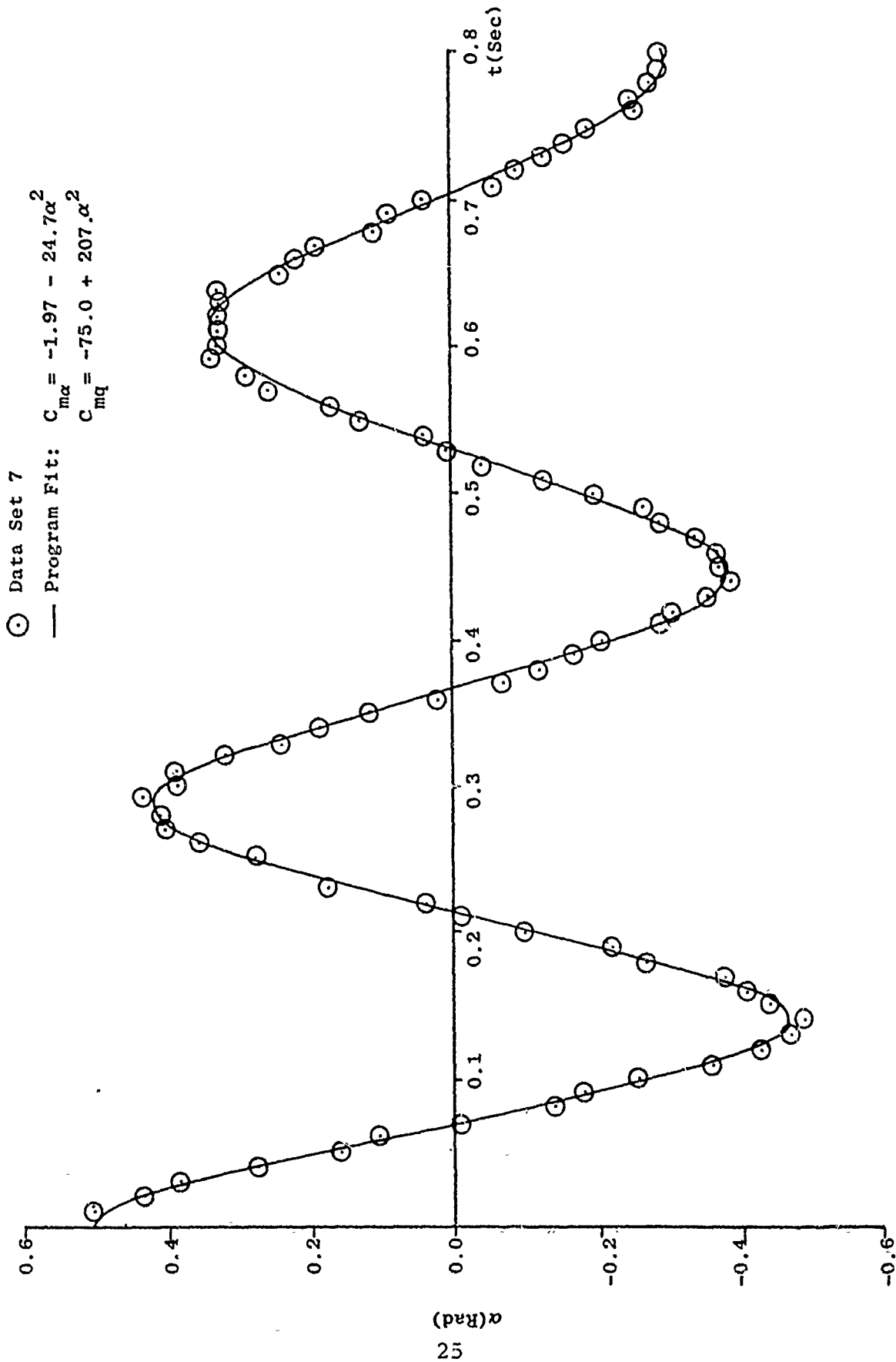
Figure 1. Example of Program Fit to Data Containing Measurement Errors

⊙ Data Set 7

— Program Fit: $C_{m\alpha} = -1.97 - 24.7\alpha^2$
$C_{mq} = -75.0 + 207.\alpha^2$

25
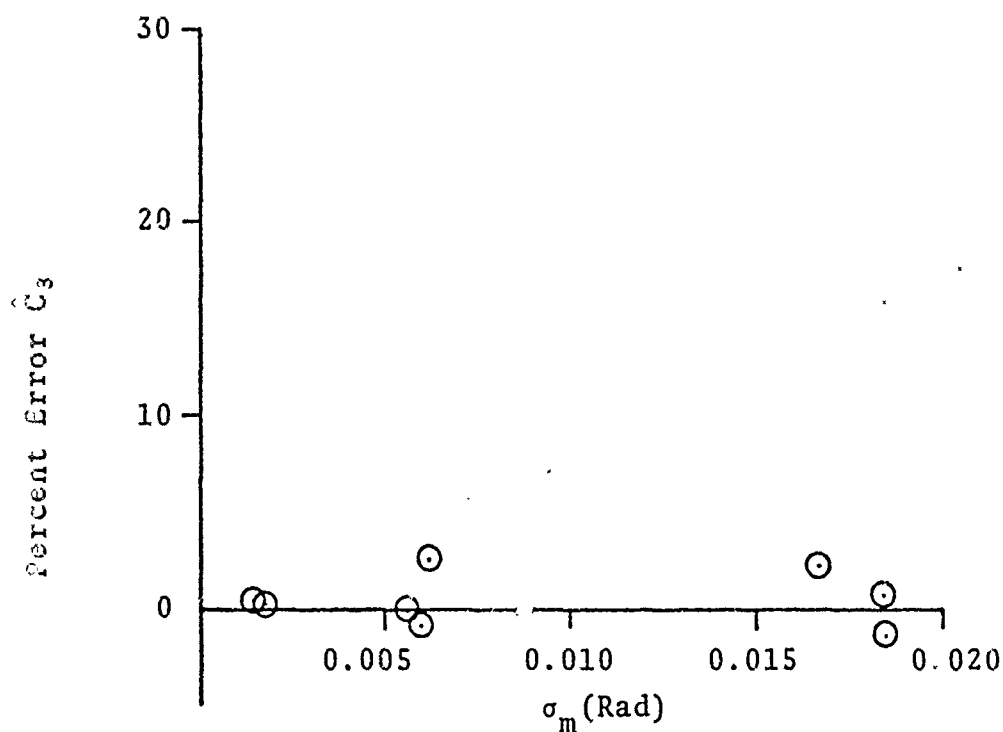
Figure 2. Variation of Percent Error in Extracted C_3 with Measurement Error
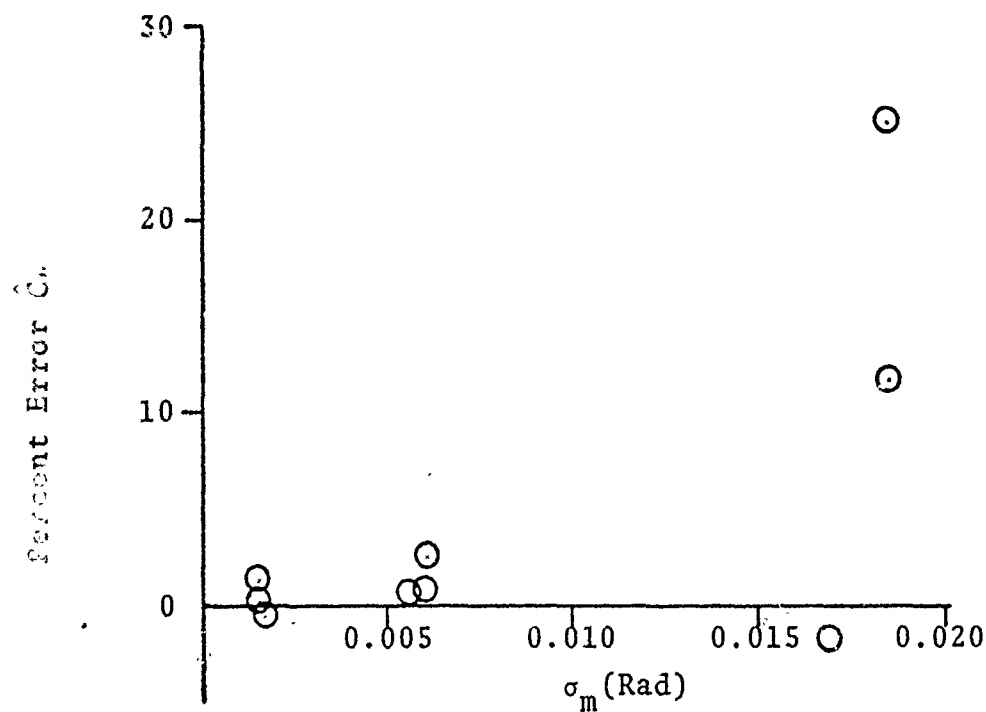
Figure 3. Variation of Percent Error in Extracted C_4 with Measurement Error
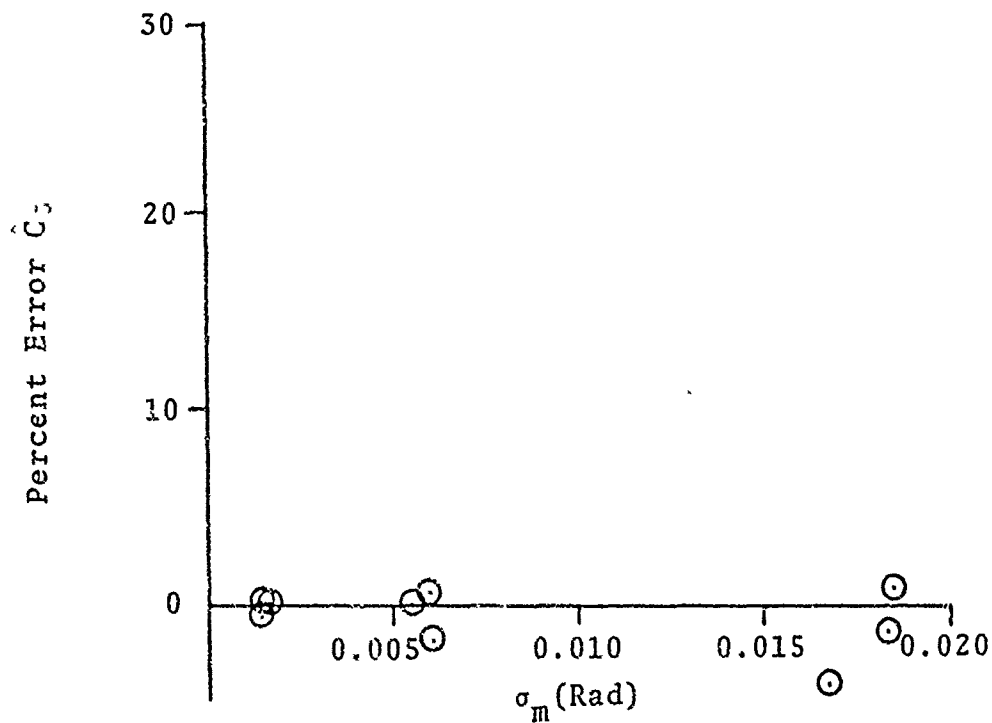
Figure 4. Variation of Percent Error in Extracted C_5 with Measurement Error
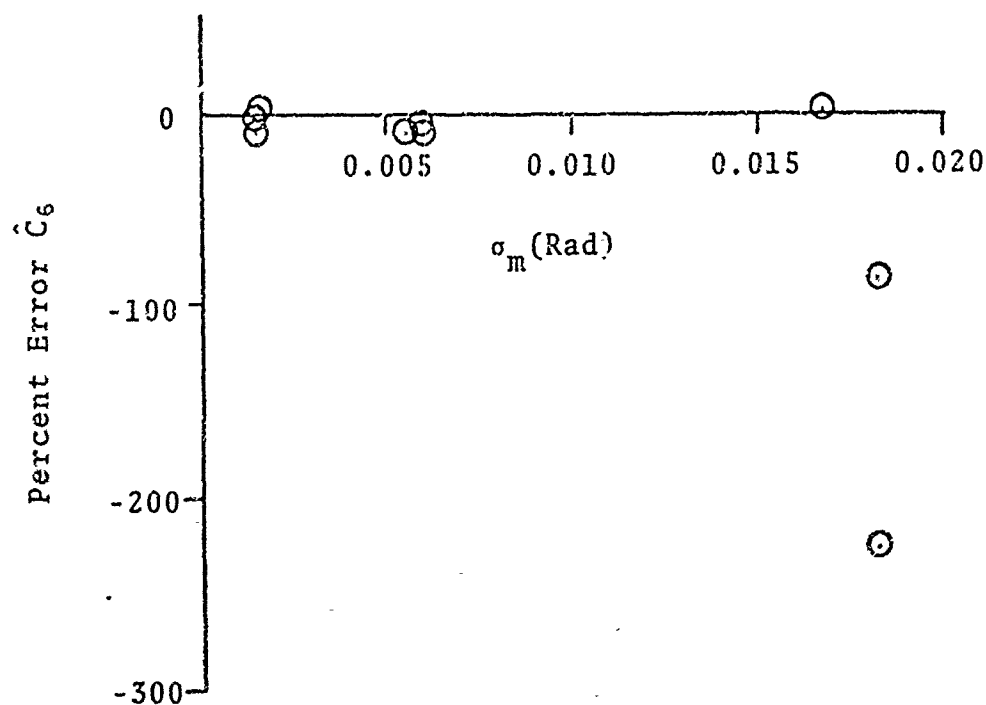
Figure 5. Variation of Percent Error in Extracted C_6 with Measurement Error

27
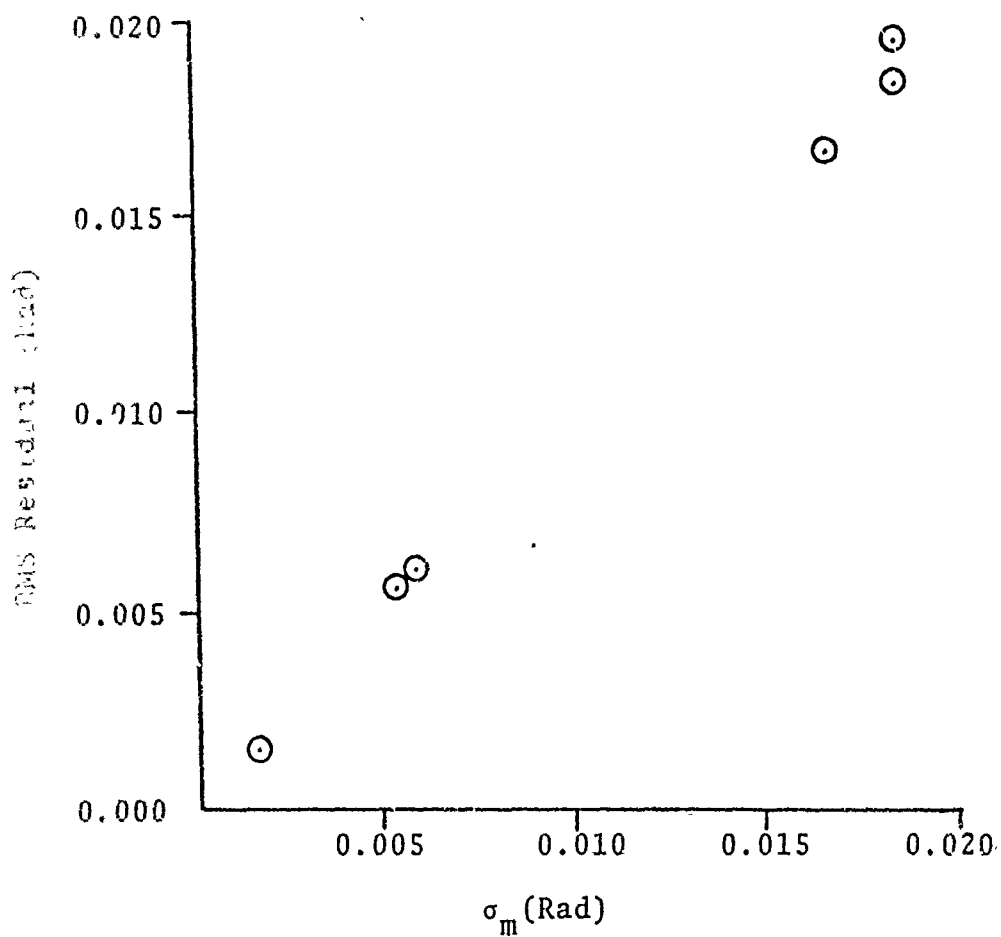
Figure 6.   Variation of RMS Residual with Measurement Error
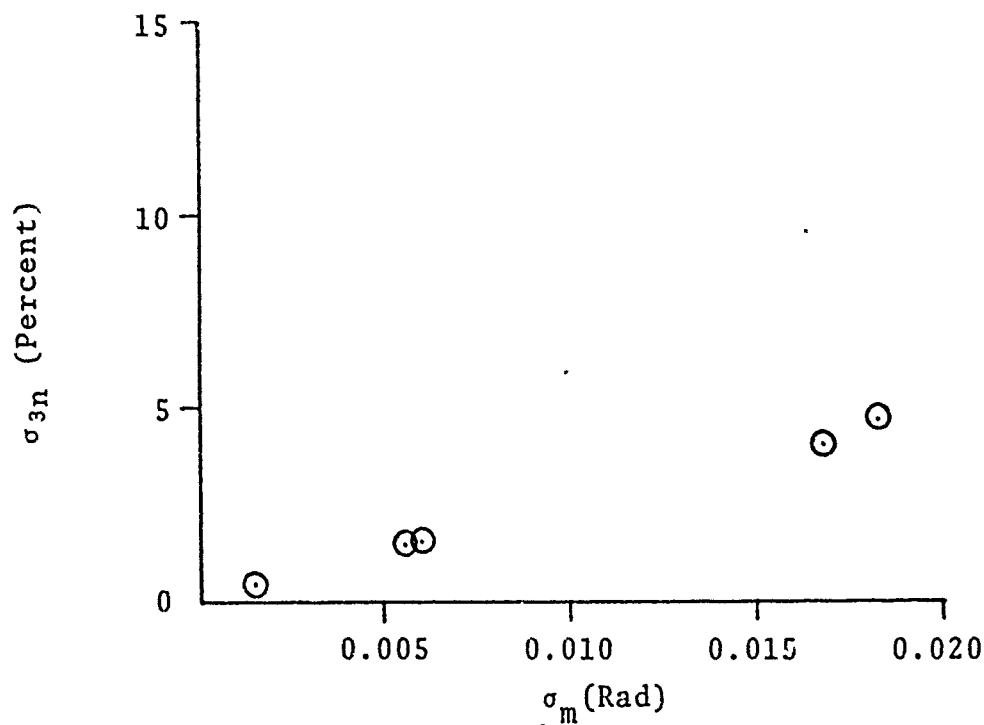
Figure 7. Variation of Normalized Estimated Standard
Deviation of $\hat{C}_3$ with Measurement Error
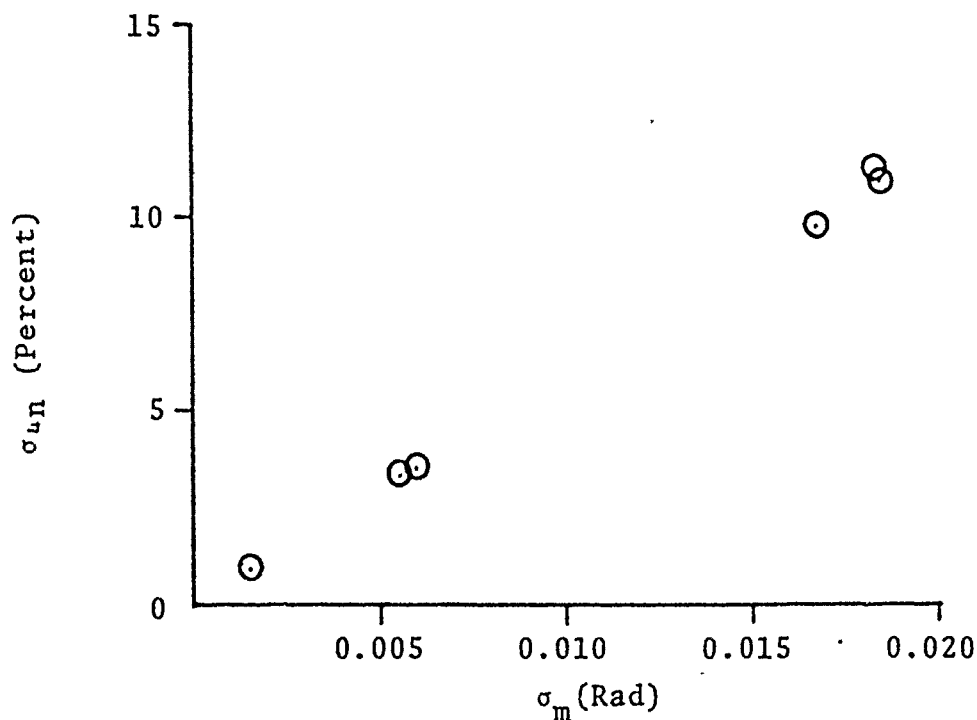
Figure 8. Variation of Normalized Estimated Standard
Deviation of $\hat{C}_4$ with Measurement Error

29
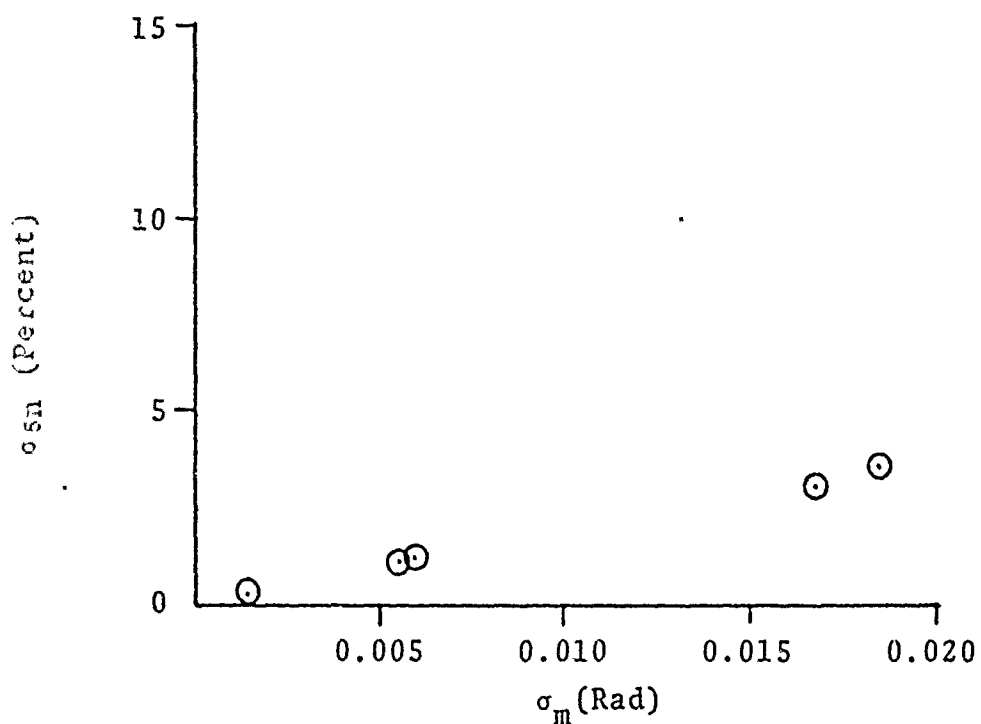
Figure 9. Variation of Normalized Estimated Standard
Deviation of $\hat{C}_5$ with Measurement Error
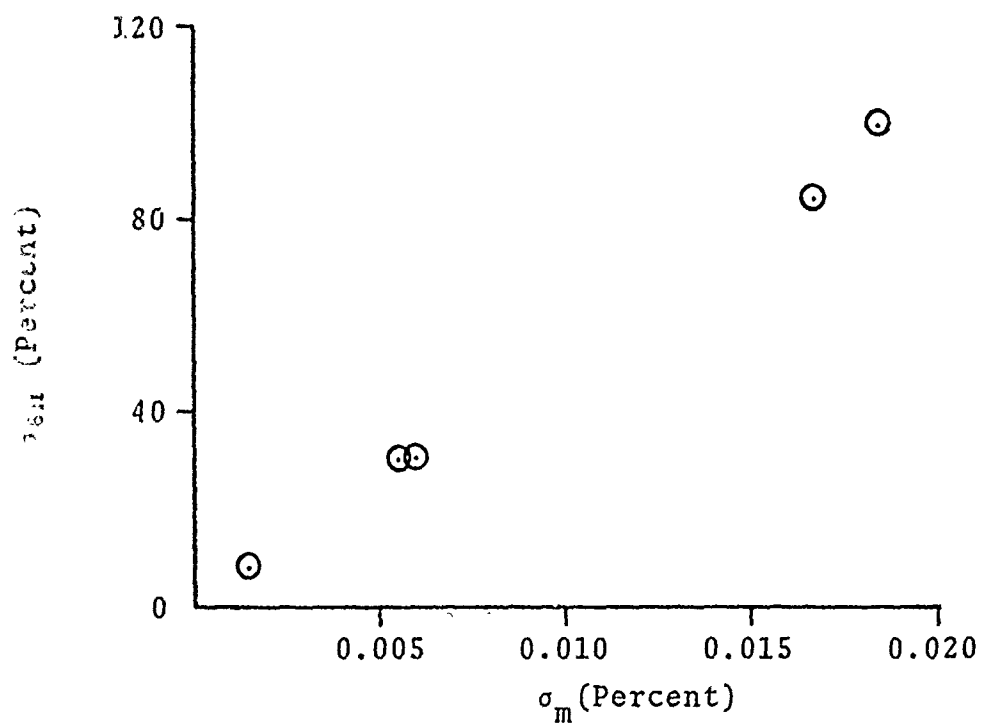
Figure 10. Variation of Normalized Estimated Standard
Deviation of $\hat{C}_6$ with Measurement Error

30
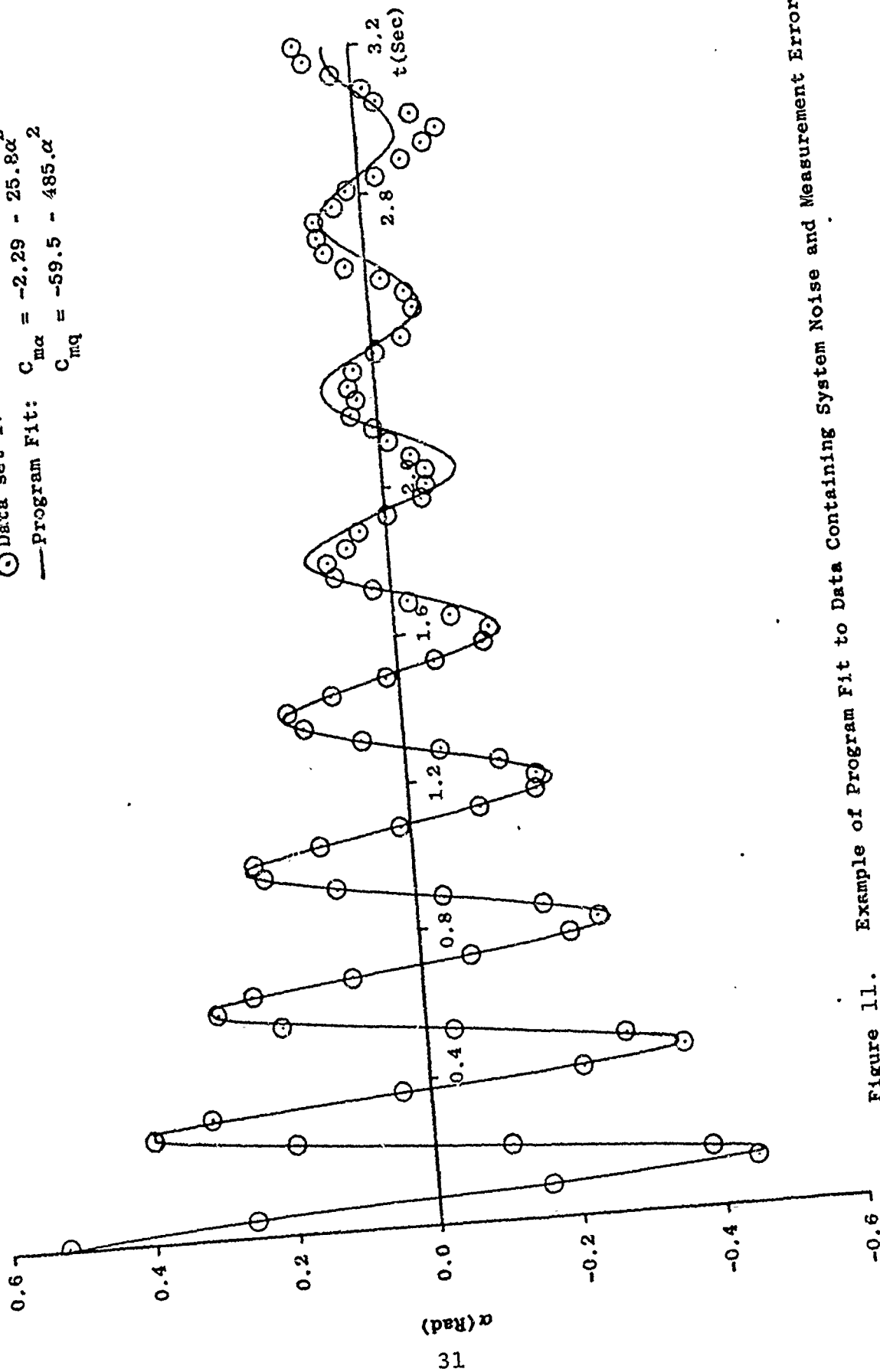
Figure 11. Example of Program Fit to Data Containing System Noise and Measurement Errors

31
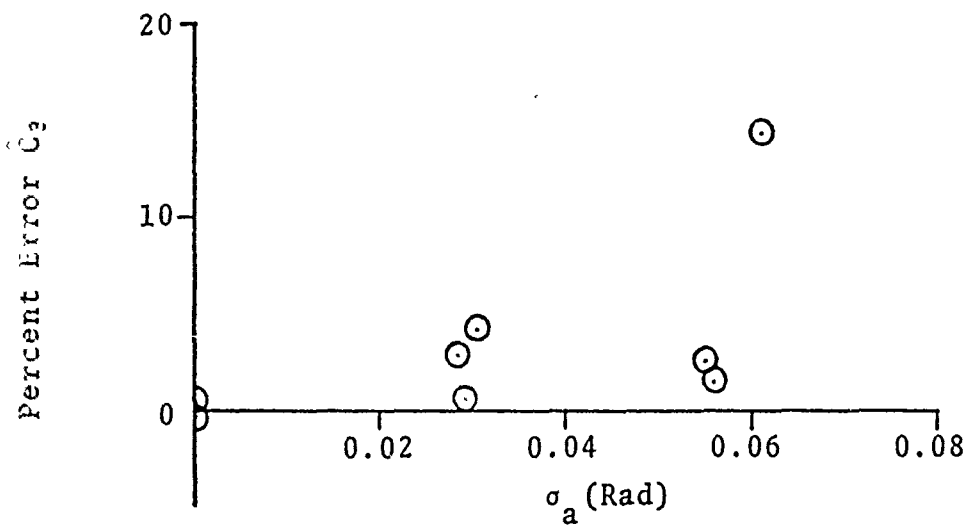
Figure 12.   Variation of Percent Error in Extracted C_3
with System Noise
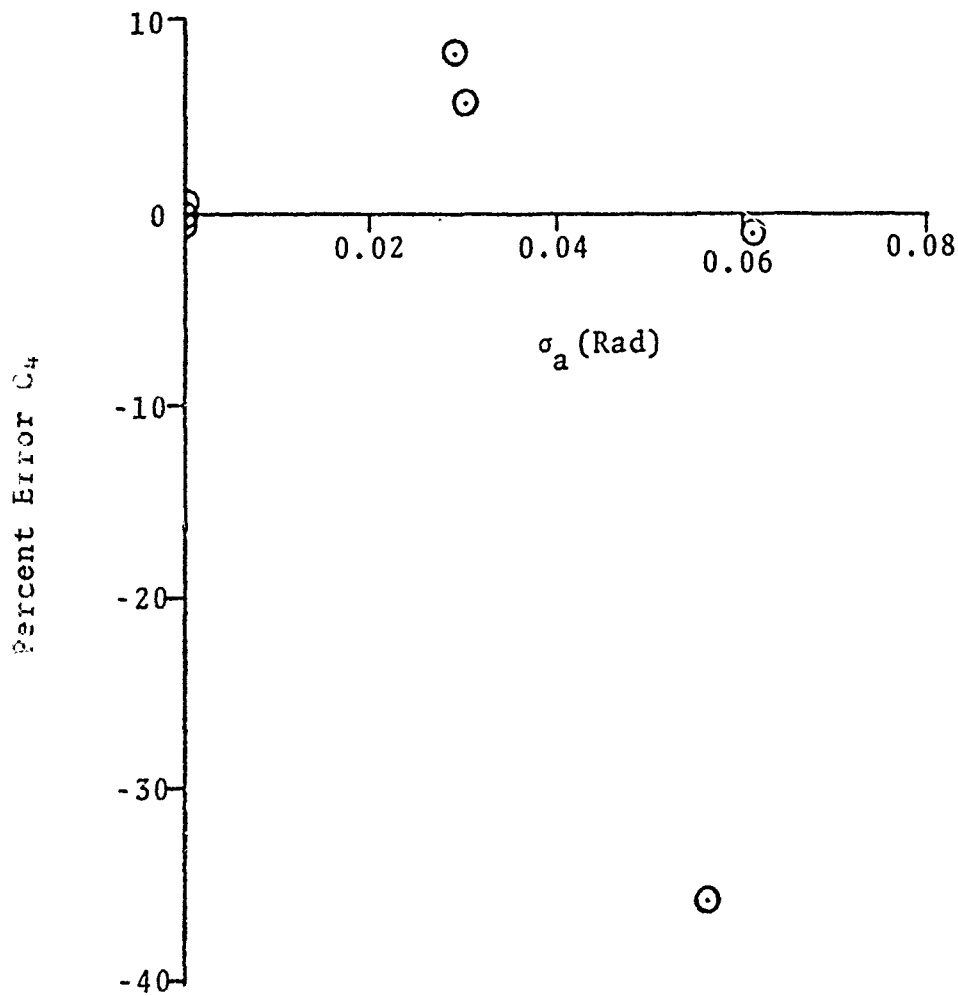
Figure 13.   Variation of Percent Error in Extracted C_4
with System Noise
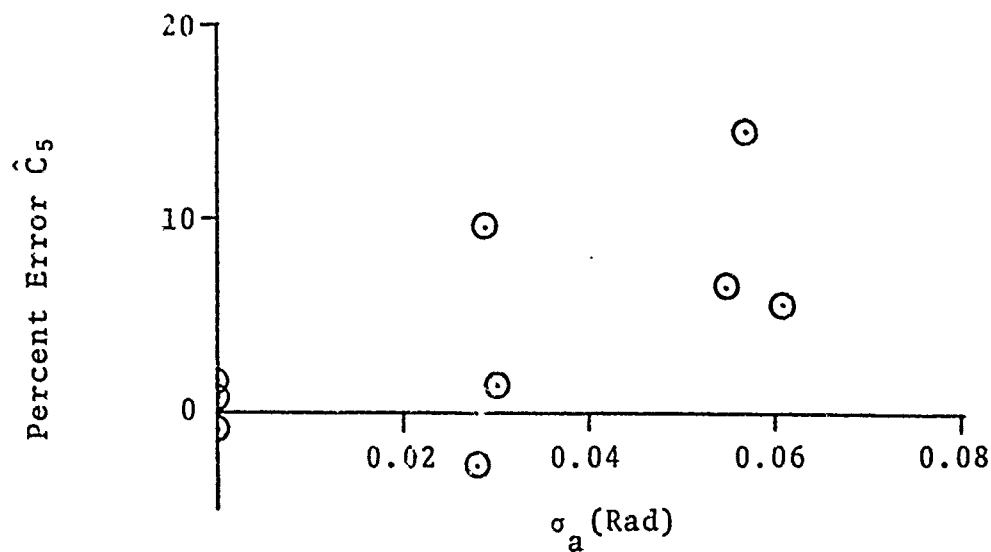
Figure 14. Variation of Percent Error in Extracted C_5
with System Noise
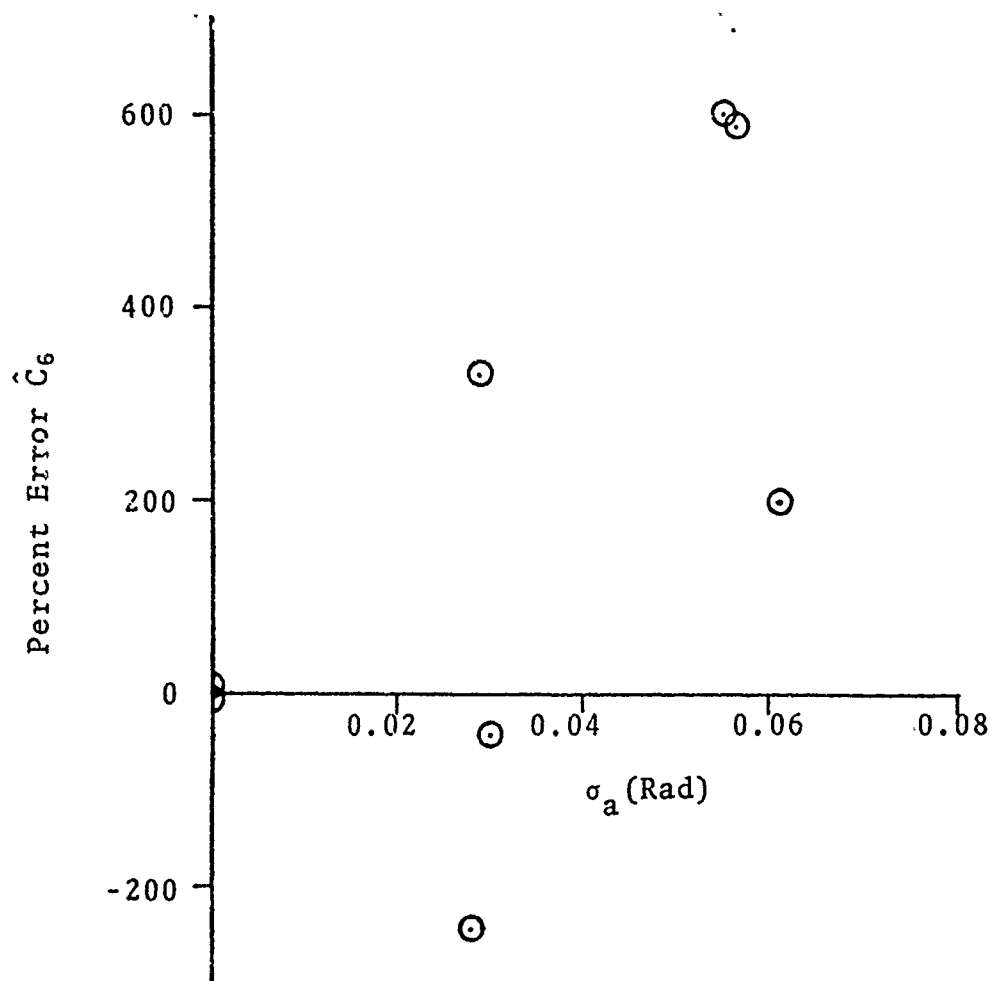
Figure 15. Variation of Percent Error in Extracted C_6
with System Noise

33
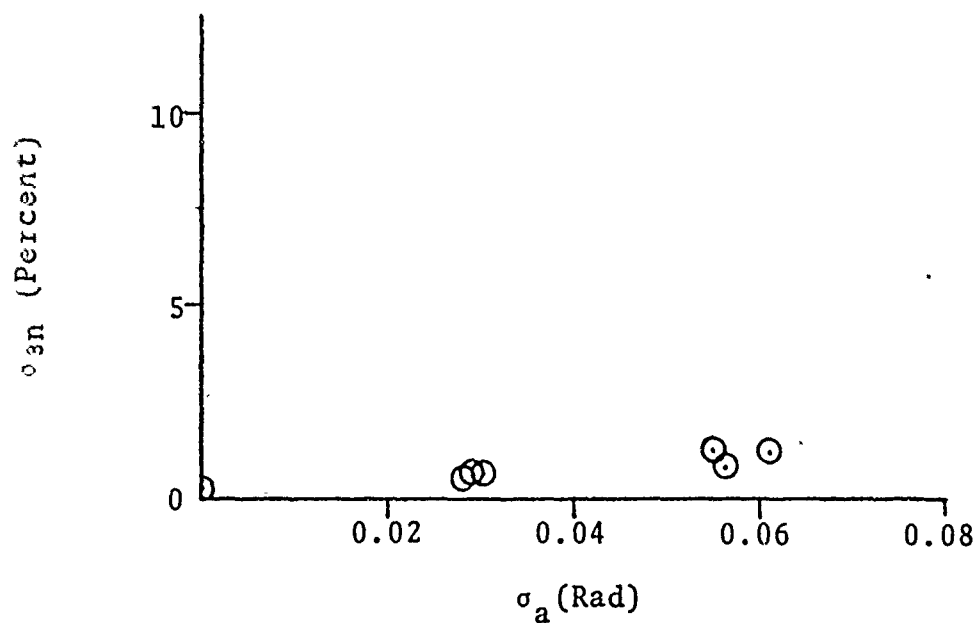
Figure 16. Variation of Normalized Estimated Standard
Deviation of $\hat{C}_3$ with System Noise
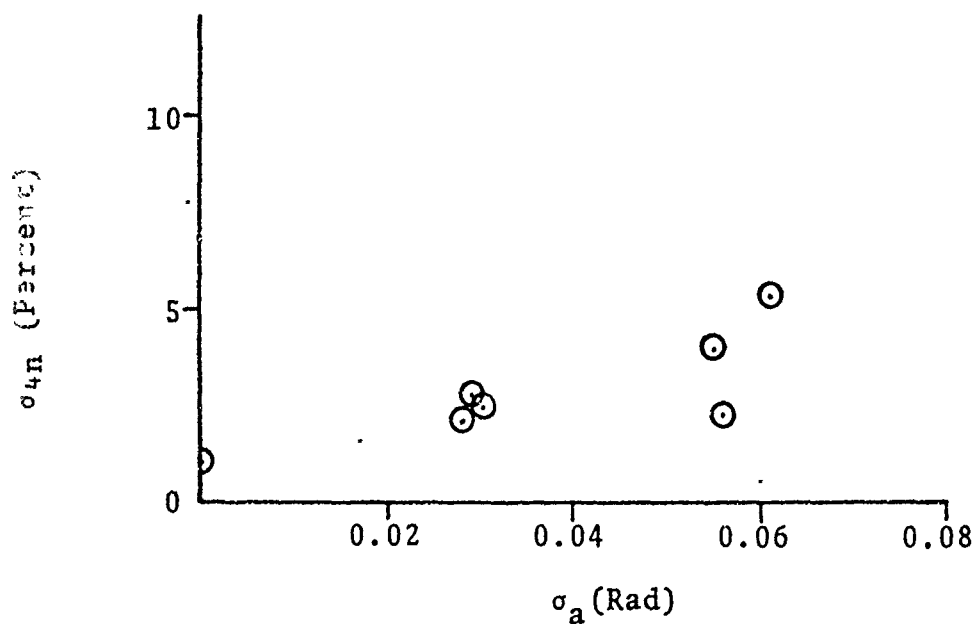
Figure 17. Variation of Normalized Estimated Standard
Deviation of $\hat{C}_4$ with System Noise
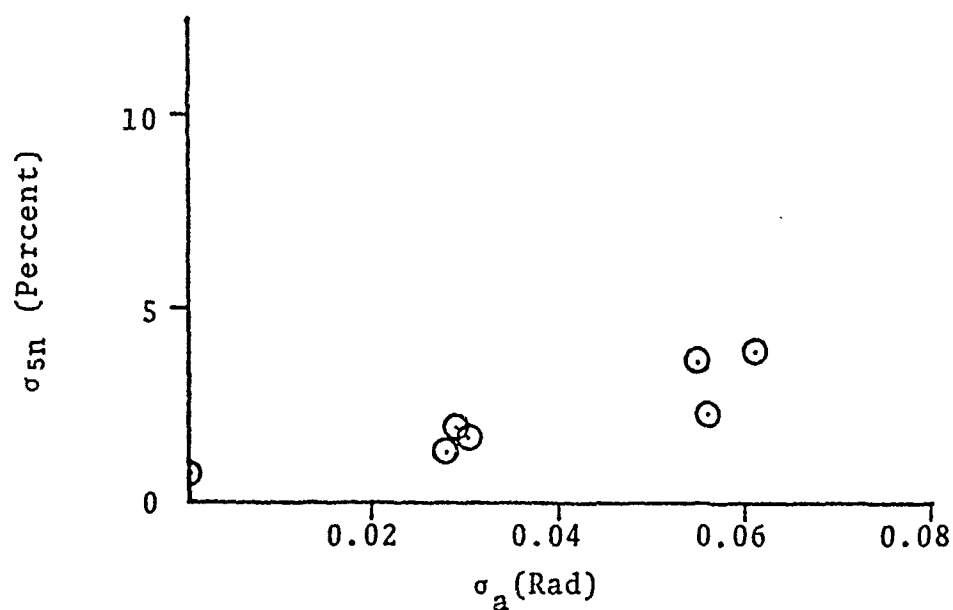
Figure 18. Variation of Normalized Estimated Standard
Deviation of $\hat{C}_5$ with System Noise
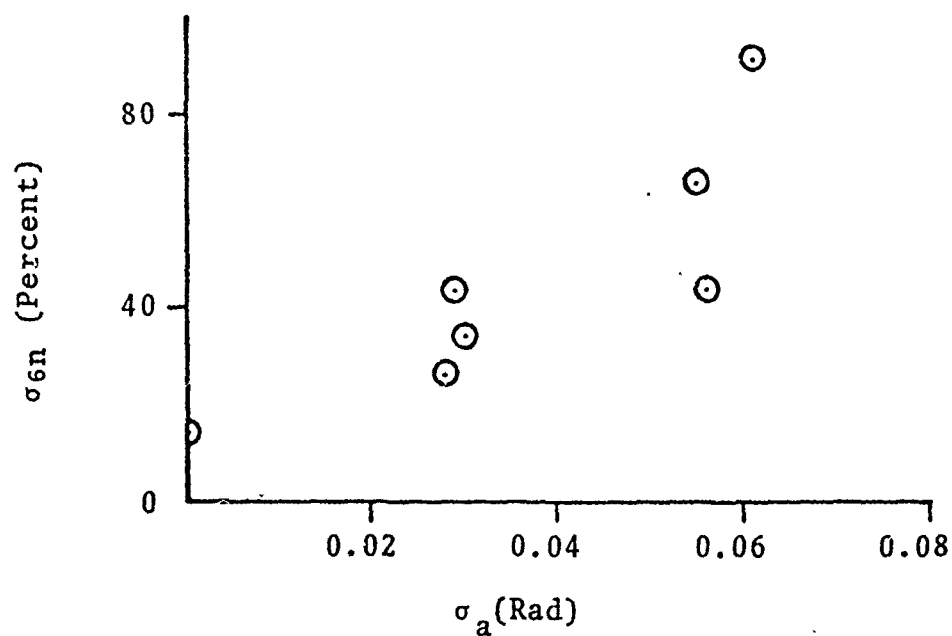
Figure 19. Variation of Normalized Estimated Standard
Deviation of $\hat{C}_6$ with System Noise

35

SECTION IV

DEVELOPMENT OF THE EXTENDED KALMAN FILTER FOR ESTIMATING PARAMETERS AND THEIR UNCERTAINTIES

1. Introduction

This chapter presents an abridged derivation of the Kalman filter for discrete and continuous linear systems, followed by a general statement of the extended Kalman filter for continuous nonlinear systems. The extended filter is then used as a base for the development of a specific algorithm for estimating states and parameters of the second order equation of pitching motion for a missile being forced by random disturbances.

2. Development of the Kalman Filter

The original derivation of the Kalman filter was presented by Kalman[18] in 1960·for multistage systems making discrete linear transitions from one stage to another. Kalman and Bucy[19] gave the analogous development for continuous linear systems approximately one year after the first work was published. The purpose of the linear filter is to provide estimates of the state of a system by making use of measurements of all, or some, of the state vector components of the system. The system is assumed to be operating in the presence of random disturbances, the statistical properties (i.e., mean and variance) of which are known. The measurements of the state vector components of the system are also assumed to have random errors of known statistics.

In addition to the original derivations of Kalman and Bucy, several other methods offering various degrees of insight but leading to the same results are available. A brief development taken primarily from Bryson and Ho[20] is presented here. Other derivations or developments of the filter equations are given by Jazwinsky,[31] Sorenson,[32] and Barham.[33]

The treatment given here starts with a static system and is extended for a single-stage linear transition which leads directly to linear multistage process. Finally, by making use of a limiting process, the desired form of the equations for a continuous linear dynamic system is given.

a. Static System

The problem at hand is to estimate the n-component state vector X of a static system using the p-component measurement vector, z, containing random errors, v, which are independent of the state. The measurement vector can be represented as

36

$$z = HX + v \qquad , \qquad (21)$$

where H is a known p x n matrix. Conditions on the measurement errors are

$$E(v) = 0 \qquad (22)$$

$$E(vv^T) = R \qquad , \qquad (23)$$

where R is a known matrix of dimension p. It is assumed that a prior estimate of the state, designated as $\bar{X}$, is available and also that the covariance of the prior estimate, M, is known. Thus

$$M = E[(X-\bar{X})(X-\bar{X})^T] \qquad , \qquad (24)$$

where M is of dimension n.

The desired estimate of X, taking into account the measurement, z, is the weighted-least-squares estimate, $\hat{X}$, which minimizes

$$J = [(X-\bar{X})^T M^{-1} (X-\bar{X}) + (z-HX)^T R^{-1} (z-HX)]. \qquad (25)$$

Differentiating Equation (25) with respect to X, setting the resulting equation to zero, and solving for X yields

$$\hat{X} = \bar{X} + PH^T R^{-1} (z-H\bar{X}) \qquad , \qquad (26)$$

where

$$P^{-1} = M^{-1} + H^T R^{-1} H \qquad . \qquad (27)$$

37

The quantity P is the covariance matrix of the error in the state estimate after measurement, X, i.e.,

$$P = E[(X-\hat{X})(X-\hat{X})^T] \qquad . \qquad (28)$$

b. Single-Stage Transitions

It is now desired to estimate the state of a system that makes a discrete transition from state 0 to state 1 according to the linear relation

$$X_1 = \phi_o X_o + \Gamma_o w_o \qquad , \qquad (29)$$

where ϕ_o is a known transition matrix of dimension n and Γ_o is a known n x r matrix. The mean and variance of the random forcing vector are assumed to be known and are given by

$$E(w_o) = \bar{w}_o \quad , \quad E[(w_o - \bar{w}_o)(w_o - \bar{w}_o)^T] = Q_o \qquad . \qquad (30)$$

The statistical properties of the random state vector are assumed to be known initially as

$$E(X_o) = \hat{X}_o \quad , \quad E[(\hat{X}_o - X_o)(\hat{X}_o - X_o)^T] = P_o \qquad . \qquad (31)$$

It is also assumed that X_o and w_o are independent. From this information X_1 is a random vector whose mean value and covariance are

$$\bar{X}_1 = \phi_o \hat{X}_o + \Gamma_o \bar{w}_o \qquad (32)$$

$$M_1 = \phi_o P_o \phi_o^T + \Gamma_o Q_o \Gamma_o^T \qquad . \qquad (33)$$

Suppose now that measurements of the state are made after the transition to state 1; then from Equations (26) and (27) the best estimate of X_1 is

$$\hat{X}_1 = \bar{X}_1 + P_1 H_1^T R^{-1} (z - H_1 \bar{X}_1) \qquad , \qquad (34)$$

38

where

$$P_1 = (M_1^{-1} + H_1^T R^{-1} H_1)^{-1}$$

$$= M_1 - M_1 H_1^T (H_1 M_1 H_1^T + R_1^{-1}) H_1 M_1 \qquad . \qquad (35)$$

c. Linear Multistage Processes

The developments of the two preceding subsections can be extended for linear, stochastic, multistage processes. Given the following difference equation system model

$$X_{i+1} = \phi_i X_i + \Gamma_i w_i \qquad , \qquad (36)$$

where

$$E(X_0) = \bar{X}_0 \qquad (37)$$

$$E(w_i) = \bar{w}_i \qquad (38)$$

$$E[(X_0 - \bar{X}_0)(X_0 - \bar{X}_0)^T] = M_0 \qquad (39)$$

$$E[(w_i - \bar{w}_i)(w_j - \bar{w}_j)] = Q_i \delta_{ij} \qquad (40)$$

$$E[(w_i - \bar{w}_i)(X_0 - \bar{X}_0)^T] = 0 \qquad (41)$$

and measurements of the state

$$z_i = H_i X_i + v_i \qquad , \qquad (42)$$

39

where

$$E(v_i) = 0 \qquad\qquad (43)$$

$$E(v_i v_j^T) = R_i \delta_{ij} \qquad\qquad (44)$$

$$E[(w_i - \bar{w}_i) v_j^T] = 0 \qquad , \qquad E[(X_o - \bar{X}_o) v_i^T] = 0 \qquad , \qquad (45)$$

the estimate of the state is

$$X_i = \bar{X}_i + K_i(z_i - H_i \bar{X}_i) \qquad , \qquad\qquad (46)$$

where

$$\bar{X}_{i+1} + \phi_i \hat{X}_i + \Gamma_i \bar{w}_i \qquad\qquad (47)$$

$$K_i = P_i H_i^T R_i^{-1} \qquad\qquad (48)$$

$$P_i = (M_i^{-1} + H_i^T R_i^{-1} H_i)^{-1}$$

$$= M_i - M_i H_i^T (H_i M_i H_i^T + R_i^{-1}) H_i M_i \qquad\qquad (49)$$

$$M_{i+1} = \phi_i P_i \phi_i^T + \Gamma_i Q_i \Gamma_i^T \qquad\qquad . \qquad\qquad (50)$$

Equations (46), (47) and (48) are the discrete Kalman filter with the state variable covariances given by Equations (49) and (50). As can be seen from the above equations, the Kalman filter is essentially the same as the system model [Equation (36)]. The differences are (1) the actual system noise, which is random from one stage to the next in Equation (36), is replaced by its mean or expected value, and (2) there is a correction term based on the difference between the actual measurement of the state and its predicted value. The difference term is multiplied by a gain, K_i, which is essentially the ratio of the uncertainty in the state to the uncertainty in the measurement. If the covariances of the measurement errors are large, the gain will be relatively small and the corresponding difference term will have little effect on the estimate of the state; if, however, the system noise is relatively large or the measurement errors are very small, the gain will be large and differences between the actual and predicted measurements of the state at a given stage have increased significance in the state variable estimates.

d. The Continuous Kalman Filter

By applying a limiting process to the discrete filter with the time between stages tending to zero, the linear system model becomes

$$\dot{X} = F(t)X + G(t)w(t) \qquad , \qquad (51)$$

and the continuous Kalman filter is given by

$$\dot{\hat{X}} = F\hat{X} + G\bar{w} + PH^T R^{-1}(z-H\hat{X}) \qquad (52)$$

$$\dot{P} = FP + PF^T + GQG^T - PH^T R^{-1}HP \qquad . \qquad (53)$$

3. The Extended Kalman Filter

The extension of the Kalman filter for estimating the states of nonlinear systems in the presence of noise has been given by Bryson and Ho[20] and Jazwinski,[31] among others, as

$$\dot{\hat{X}} = f(\hat{X},t) + G(t)\bar{w}(t) + P\left[\frac{\partial h}{\partial X}\right]^T R^{-1}[z(t)-h(\hat{X},t)] \qquad (54)$$

$$\dot{P} = \frac{\partial f}{\partial X}P + P\left[\frac{\partial f}{\partial X}\right]^T + GQG^T - P\left[\frac{\partial h}{\partial X}\right]^T R^{-1}\left[\frac{\partial h}{\partial X}\right]P \qquad (55)$$

for the nonlinear system

$$\dot{X} = f(X,t) + G(t)w(t) \qquad (56)$$

with measurements

$$z(t) = h(X,t) + v(t) \qquad , \qquad (57)$$

where

$$E[w(t)] = \bar{w}(t) \qquad (58)$$

$$E[w(t)-\bar{w}(t)][w(t')] = Q(t)\delta(t-t') \qquad (59)$$

$$E[X(t_o)] = \bar{X}_o \qquad (60)$$

$$E[X(t_o)-\bar{X}_o][X(t_o)-\bar{X}_o]^T = P_o \qquad (61)$$

$$E[X(t_o)-\bar{X}_o][w(t)-\bar{w}(t)]^T = 0 \qquad (62)$$

$$E[v(t)] = 0 \qquad (63)$$

$$E[v(t)v^T(t')] = R(t)\delta(t-t') \qquad (64)$$

42

$$E[X(t_o) - \bar{X}_o][v(t)]^T = 0 \tag{65}$$

$$E[w(t) - \bar{w}(t)][v(t')]^T = 0 \tag{66}$$

4. The Parameter Estimation Algorithm

In this subsection a specific parameter and state estimation algorithm using the extended Kalman filter is developed for the basic equation of pitching motion previously analyzed with the Chapman-Kirk technique.

The nonlinear equation of motion, or system model, is thus

$$\ddot{\alpha} + (C_4 + C_6\alpha^2)\dot{\alpha} + (C_3 + C_5\alpha^2)\alpha = w(t) \tag{67}$$

where

$$E[w(t)] = 0 \quad , \quad E[w(t)w(t')] = q\delta(t-t') \quad . \tag{68}$$

The measurements of the state of the system are assumed to be given by

$$z(t) = \alpha(t) + v(t) \tag{69}$$

where

$$E[v(t)] = 0 \quad , \quad E[v(t)v^T(t')] = r\delta(t-t') \quad . \tag{70}$$

Now to reduce Equation (67) to the required system of first order differential equations, let

43

$$\alpha = X_1$$

$$\dot{\alpha} = X_2 \qquad \qquad . \qquad (71)$$

At this point, to estimate the aerodynamic parameters appearing in the equation of motion in addition to the state variables, the state vector is augmented by setting

$$C_3 = X_3$$

$$C_4 = X_4$$

$$C_5 = X_5$$

$$C_6 = X_6 \qquad (72)$$

with the constraints

$$\dot{X}_3 = 0$$

$$\dot{X}_4 = 0$$

$$\dot{X}_5 = 0$$

$$\dot{X}_6 = 0 \qquad \qquad . \qquad (73)$$

Making use of Equations (71), (72), and (73), Equation (67) now reduces to the following nonlinear system of first order equations:

$$\dot{X}_1 = X_2$$

44

$$\dot{X}_2 = -(X_4 + X_6 X_1{}^2) X_2 - (X_3 + X_5 X_1{}^2) X_1 + w(t)$$

$$\dot{X}_3 = 0$$

$$\dot{X}_4 = 0$$

$$\dot{X}_5 = 0$$

$$\dot{X}_6 = 0 \qquad , \tag{74}$$

with linear measurements

$$z = X_1 + v \qquad . \tag{75}$$

Comparing Equations (74) and (75) with Equations (56) and (57), the following matrices may be identified:

$$G(t) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad H(t) = \begin{bmatrix} X_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$f(X,t) = \begin{bmatrix} X_2 \\ -(X_4 + X_6 X_1{}^2) X_2 - (X_3 + X_5 X_1{}^2) X_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad . \tag{76}$$

45

Now, since the mean value of the system noise is assumed to be zero [Equation (68)] and because the measurements of the state are related linearly to the state [Equation (75)], the extended Kalman filter, previously given by Equations (54) and (55), can be simplified somewhat to

$$\dot{\hat{X}} = (\hat{X}, t) + PH^T \left[\frac{1}{r}\right] [z - H\hat{X}] \qquad (77)$$

$$P = \frac{\partial f}{\partial X} P + P\left[\frac{\partial f}{\partial X}\right]^T + GqG^T - PH^T\left[\frac{1}{r}\right]HP \quad , \qquad (78)$$

where

$$H = [\ 1\ 0\ 0\ 0\ 0\ 0\ ] \qquad .$$

Applying Equations (76) and (77) yields

$$
\begin{bmatrix} \dot{\hat{X}}_1 \\ \dot{\hat{X}}_2 \\ \dot{\hat{X}}_3 \\ \dot{\hat{X}}_4 \\ \dot{X}_5 \\ \dot{\hat{X}}_6 \end{bmatrix} = \begin{pmatrix} X_2 \\ -(X_4 + X_6 X_1{}^2) X_2 - (X_3 + X_5 X_1{}^2) X_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
$$

$$
+ \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} \\ P_{12} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} \\ P_{13} & P_{23} & P_{33} & P_{34} & P_{35} & P_{36} \\ P_{14} & P_{24} & P_{34} & P_{44} & P_{45} & P_{46} \\ P_{15} & P_{25} & P_{35} & P_{45} & P_{55} & P_{56} \\ P_{16} & P_{26} & P_{36} & P_{46} & P_{56} & P_{66} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \frac{1}{r}(z - [1\ 0\ 0\ 0\ 0\ 0] \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \hat{X}_3 \\ \hat{X}_4 \\ \hat{X}_5 \\ \hat{X}_6 \end{bmatrix}) \quad .
$$

46

Carrying out the prescribed multiplications and equating like components of the resulting two column matrices yields the desired filter equations,

$$\dot{\hat{X}}_1 = \hat{X}_2 + \frac{P_{11}}{r} (z-\hat{X}_1)$$

$$\dot{\hat{X}}_2 = -(\hat{X}_4+\hat{X}_6\hat{X}_1^2)\hat{X}_2 - (\hat{X}_3+\hat{X}_5\hat{X}_1^2)\hat{X}_1 + \frac{P_{12}}{r} (z-\hat{X}_1)$$

$$\dot{\hat{X}}_3 = \frac{P_{13}}{r} (z-\hat{X}_1)$$

$$\dot{\hat{X}}_4 = \frac{P_{14}}{r} (z-\hat{X}_1)$$

$$\dot{\hat{X}}_5 = \frac{P_{15}}{r} (z-\hat{X}_1)$$

$$\dot{\hat{X}}_6 = \frac{P_{16}}{r} (z-\hat{X}_1) \qquad\qquad\qquad (79)$$

The necessary covariance equations are obtained from the matrix Ricatti equation [Equation (78)] making use of Equations (76) and the fact that

$$\left[\frac{\partial f}{\partial X}\right]_{X=\hat{X}} =$$

$$\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-2\hat{X}_1\hat{X}_2\hat{X}_6-\hat{X}_3-3\hat{X}_1^2\hat{X}_5 & -(\hat{X}_4+\hat{X}_6\hat{X}_1^2) & -\hat{X}_1 & -\hat{X}_2 & -\hat{X}_1^3 & -\hat{X}_1^2\hat{X}_2 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$(80)$$

The results of substituting Equations (76) and (80) into (78), carrying out the rather tedious matrix multiplications, and equating like terms are

$$\dot{P}_{11} = 2P_{12} - \frac{1}{r}P_{11}{}^2$$

$$\dot{P}_{12} = P_{22} - AP_{11} - BP_{12} - \hat{X}_1 P_{13} - \hat{X}_2 P_{14} - CP_{15} - DP_{16} - \frac{1}{r}P_{11}P_{12}$$

$$\dot{P}_{13} = P_{23} - \frac{1}{r}P_{11}P_{13}$$

$$\dot{P}_{14} = P_{24} - \frac{1}{r}P_{11}P_{14}$$

$$\dot{P}_{15} = P_{25} - \frac{1}{r}P_{11}P_{15}$$

$$\dot{P}_{16} = P_{26} - \frac{1}{r}P_{11}P_{16}$$

$$\dot{P}_{22} = q - 2(AP_{12} + BP_{22} + \hat{X}_1 P_{23} + \hat{X}_2 P_{24} + CP_{25} + DP_{26}) - \frac{1}{r}P_{12}{}^2$$

$$\dot{P}_{23} = -AP_{13} - BP_{23} - \hat{X}_1 P_{33} - \hat{X}_2 P_{34} - CP_{35} - DP_{36} - \frac{1}{r}P_{12}P_{13}$$

$$\dot{P}_{24} = -AP_{14} - BP_{24} - \hat{X}_1 P_{34} - \hat{X}_2 P_{44} - CP_{45} - DP_{46} - \frac{1}{r}P_{12}P_{14}$$

$$\dot{P}_{25} = -AP_{15} - BP_{25} - \hat{X}_1 P_{35} - \hat{X}_2 P_{45} - CP_{55} - DP_{56} - \frac{1}{r}P_{12}P_{15}$$

$$\dot{P}_{26} = -AP_{16} - BP_{26} - \hat{X}_1 P_{36} - \hat{X}_2 P_{46} - CP_{56} - DP_{66} - \frac{1}{r}P_{12}P_{16}$$

$$\dot{P}_{33} = -\frac{1}{r}P_{13}{}^2$$

$$\dot{P}_{34} = -\frac{1}{r}P_{13}P_{14}$$

$$\dot{P}_{35} = -\frac{1}{r}P_{13}P_{15}$$

$$\dot{P}_{36} = -\frac{1}{r}P_{13}P_{16}$$

$$\dot{P}_{44} = -\frac{1}{r}P_{14}{}^2$$

$$\dot{P}_{45} = -\frac{1}{r}P_{14}P_{15}$$

$$\dot{P}_{46} - -\frac{1}{r}P_{14}P_{16}$$

$$\dot{P}_{55} = -\frac{1}{r}P_{15}{}^2$$

$$\dot{P}_{56} = -\frac{1}{r}P_{15}P_{16}$$

$$\dot{P}_{66} = -\frac{1}{r}P_{16}{}^2$$

where

$$A = 2\hat{X}_1\hat{X}_2\hat{X}_6 + \hat{X}_3 + 3\hat{X}_1^2\hat{X}_5$$

$$B = \hat{X}_4 + \hat{X}_6\hat{X}_1^2$$

$$C = \hat{X}_1^3$$

$$D = \hat{X}_1^2\hat{X}_2 \qquad\qquad (82)$$

The desired estimates of the states and parameters are obtained by numerically integrating the filter equations [Equations (79)] and their covariances [Equations (81)]. Initial estimates of the states and their covariances as well as the variances of the system noise and measurement errors are assumed to be available. In the event that the data consist of discrete measurements, which is usually the case, the constant time between data points should be equal to the numerical integration step size. This is necessary because the state estimates are updated at each integration increment and, in doing this, the filter requires knowledge of the difference between measured and estimated state values.

If the filter successfully adjusts the true states and the imbedded parameters so that X_1 is an adequate match to the data, z, the time derivatives of the filter equations for the parameters become small [see Equations (79)] and the parameters reach steady-state or near steady-state conditions.

USE OF THE EXTENDED KALMAN FILTER FOR
ESTIMATING PARAMETERS AND THEIR UNCERTAINTIES

1.   Introduction

This section presents an analysis using the extended
Kalman filter with parameter augmented state vector to
determine aerodynamic coefficients and their uncertainties
from noisy dynamic data. A description of the digital
computer program used in the analysis is given first,
followed by results obtained with this program from a
variety of noisy data sets.

2.   The Extended Kalman Filter Program

The extended Kalman filter program is written in
Fortran IV for use primarily on an IBM model 360/65
digital computer. The basic function of the program is
to integrate numerically the 27 first order differential
equations which are given in the previous chapter and
which comprise the extended Kalman filter with parameter
augmented state vector. The specific functions of the
main program and its subroutines, as well as the required
input data, are described in the following paragraphs.
A listing of the complete program is given in Appendix II.

   a. Main Program Functions

       The main program reads and writes the input data
       and calls the numerical integration subroutine,
       ADDUM.

   b. Subroutine Functions

       The names and functions of each of the three sub-
       routines that are used in the extended Kalman filter
       program are given below.

   (1) ADDUM

       Subroutine ADDUM integrates the 27 filter and
       covariance equations using a fourth order Runge-
       Kutta method for starting, followed by a fourth
       order Adams-Bashforth predictor-corrector method
       for running. It is essentially identical to the
       subroutine of the same name used in the Chapman-
       Kirk coefficient extraction program and is described
       in detail in Reference 26. Subroutines XDOT and OUT
       are called from ADDUM.

50

(2) XDOT

    Subroutine XDOT computes values of the time deri-
vatives for the differential equations being
integrated by ADDUM.

(3) OUT

    This subroutine writes the output of the numerical
integration.

c. Required Input Data

    The program reads four categories of input data.
These categories and the specific data in each are
delineated in Appendix II. The format and units
of the entries on a specific data card can be
determined from the program listing and nomenclature
list, which are also given in Appendix II.

3. Analysis of the Extended Kalman Filter

    The use of the extended Kalman filter for estimating
the parameters of interest is analyzed for linear and non-
linear systems with data containing only measurement errors
as well as data containing both system noise and measurement
errors. The analysis begins with a linear system and data
containing only measurement errors and progresses through
increasing stages of difficulty up to nonlinear systems and
data which have been corrupted by both system noise and
measurement errors. The previously discussed computer
program and algorithm require no modifications to consider
the linear case; by initially setting the parameters that
are the numerical coefficients of the nonlinear terms in
the equation of motion (and their variances) equal to zero,
the extended filter for a nonlinear system reduces to the
one required for a linear system.

    All data used in the analysis were generated by the
previously mentioned computer program, UFNOISE (Reference
29). The true values of the constants used in the data
generation are those previously given in Table III with the
exception of the initial value of the pitch angle for the
linear system. This initial displacement is a more realistic
0.1745 radian for the linear cases considered.

a. Linear Systems with Measurement Errors Only

    After the extended Kalman filter computer program
had been constructed and checked, it became apparent
that the technique would probably be best under-
stood by considering relatively simple cases at first

and then progressing to more difficult ones as
confidence in the technique was gained. To this
end, the first success with the program was rea-
lized when analyzing data containing only measurement
errors for a linear system.

The pitch angle data used in this part of the
analysis were generated by integrating the equation
of motion

$$\ddot{\alpha} + C_4\dot{\alpha} + C_3\alpha = 0 \tag{83}$$

and superimposing random Gaussian errors on the
output. The standard deviation of the errors is
0.00588 radian, which corresponds to 3σ measurement
errors of approximately 1.0 degree. The numerical
integration step size used in generating the data
was 0.005 second, and the pitch angle was output
every integration increment. The equation was in-
tegrated for a total of 2.00 seconds.

(1) Basic Filter Performance

The results of using the extended Kalman filter to
identify the correct values of the parameters of
interest, X_3 and X_4, are shown on Figures 20 and
21. These figures show time histories of the per-
cent errors in the estimated values of the parameters
that were computed by the extended filter program
when analyzing the noisy data described previously.
The percent error is defined by

$$\text{PERCENT ERROR } (\hat{X}_j) = \frac{(\hat{X}_j - X_j) \cdot 100}{X_j} \quad . \tag{84}$$

The input for the measurement error variance was the
one previously computed by the simulation program
based on the errors that were actually put into the
data. The initial values of the parameter variances
were computed from knowledge of the true value of
the parameters and the arbitrarily selected initial
values of the parameters. These sample variances
are defined by

$$P_{jj}(0) = [X_j(0) - \hat{X}_j(0)]^2 \quad . \tag{85}$$

52

This method of initializing the parameter variances was chosen merely to generate the required initial numerical values. In actual experimentation, the initial variance values would depend on the method of selecting the initial parameter estimates and prior knowledge as to how accurately these initial parameter estimates were in relation to the true values of the parameters.

The initial values of all covariances were chosen as zero. This implies that errors in the estimates of the individual state vector components are initially uncorrelated. The initial values of the variances for the pitch angle and pitch angle rate were also chosen as zero. For wind tunnel dynamic experimentation where the model is initially held rigid at some given displacement to the wind stream, this seems to be a valid assumption.

As can be seen from Figures 20 and 21, the filter does an excellent job of identifying the parameters of interest. The X_3 parameter, which corresponds to the $C_{m\alpha}$ term and which is initially in error by 25 percent, is identified with approximately zero percent error in less than 0.3 second. The correct identification of the damping parameter takes slightly more time but the results are of equal quality.

(2) Effects of Variations in the Initial Parameter Variances

This subsection presents some effects on the near steady-state estimates of the parameters of interest and their near steady-state variances for various values of the initial parameter variances. The parameter variances were initialized to values that were 25 percent higher than those used in the previous sub section and also to values that were 25 percent lower than the referenced values. The effects of these initializations are shown in Figures 22 through 25 and in Table VII for the linear system of interest.

Table VII is a summary of the percent errors in the parameter variances and the normalized parameter uncertainties for the above-mentioned variations in the initial parameter variances.

The same results are depicted graphically in Figures 22 and 23. The numerical results in Table VII are based on the near steady-state parameter and variance values that result after the filter has integrated for 1.5 seconds. These correspond to the last point

53

TABLE VII. EFFECTS OF VARIATIONS IN THE INITIAL
PARAMETER VARIANCES ON NEAR STEADY-
STATE PARAMETERS AND THEIR VARIANCES
(LINEAR SYSTEM, MEASUREMENT ERRORS ONLY)

Variation in Initial Parameter Variance* (Percent)	Errors in Near Steady-State Parameters (Percent)		Normalized Near Steady-State Variances (Percent)	
	$\hat{X}_3$	$\hat{X}_4$	$\dfrac{\sqrt{P_{33}}}{n}$	$\dfrac{\sqrt{P_{44}}}{n}$
−25	0.17	0.55	0.12	0.91
0	0.19	0.46	0.11	0.91
25	0.22	0.38	0.11	0.91

*relative to sample variance

shown on the supporting figures. The normalized parameter uncertainties are defined by

$$\text{PERCENT } (\sqrt{P_{jj}})_n = \frac{\sqrt{P_{jj}}}{X_j} \cdot 100 \qquad . \qquad (86)$$

Time histories of the parameter variances, P_{33} and P_{44}, are shown in Figures 24 and 25. All of the information presented in these figures indicates that initial values of parameter variances in the range investigated have no effect on final parameter estimates or their variances when using the extended Kalman filter to analyze data for a linear system containing only measurement errors.

b. Linear Systems with Measurement Errors and System Noise

Data for this portion of the analysis were generated with the computer program UFNOISE by numerically integrating the following equation of motion

$$\ddot{\alpha} + C_4 \dot{\alpha} + C_3 \alpha = w(t) \qquad (87)$$

and superimposing random Gaussian errors on the output. The forcing function, $w(t)$, is also random in nature and Gaussian with zero mean. The standard deviation of the system noise was 4.80 rad/sec^2, which is of sufficient magnitude to drive the oscillations in a noise band of approximately 0.0872 radian (5 degrees) for zero initial displacement. The standard deviation of the measurement errors was 0.00556 radian. Also, as before, the equation of motion was integrated in increments of 0.005 second for a total of 2.00 seconds.

(1) Basic Filter Performance

The basic performance of the filter is demonstrated in Figures 26 and 27 which show time histories of the percent error in X_3 and X_4. As can be seen from these figures, the initial estimates of X_3 and X_4 are 25 percent in error and these estimates are

55

corrected to within approximately 1 percent and
3 percent of their respective true values. The
noise variances used are those given by the
simulation which generated the pitch angle data.
The initial parameter variances are calculated
from Equation (85).

(2) Effects of Variations in the Initial Parameter
Variances

The sensitivity of the filter to variations in
the initial parameter variances is given in
Table VIII and in Figures 28 through 32. Table
VIII is a summary of the percent error in the
parameters of interest and their normalized un-
certainties for variations in the initial para-
meter variances of -25 percent, 0.0 percent and
25 percent relative to the sample variances.

Figures 28 and 29 show the variation of the near
steady-state errors in the parameters and their
normalized uncertainties as functions of the per-
cent variations in the initial parameter variances.
Figure 30 shows the two time histories of the
error in the estimate of X_4 that result for two
different initial values of the parameter variances.
One of the initial variances is the sample variance
based on the initial and true values of the para-
meter [Equation (85)], and the other is 25 percent
less than this value.

The X_3 error time histories are essentially iden-
tical for both cases and are not presented. The
variations of the parameter variances with time,
for the three initial values considered, are shown
in Figures 31 and 32.

From the information given in the figures mentioned
above, it is obvious that neither X_3 nor P_{33} are
affected by variations in the initial variances
within the ±25 percent range considered. The es-
timates of the damping parameter as well as its
variance are affected, however, by these variations.
The near steady-state damping parameter estimates
vary almost linearly with initial variance values
from a low of 0.47 percent to a high of 4.95 per-
cent (see Figure 28). The variation in the normalized
uncertainty is less pronounced, ranging from a low
of 14.39 percent to a high of 15.81 percent (see
Figure 39). Nevertheless, the near steady-state
uncertainties are of sufficient magnitude so that
the true values of the coefficients are within less

56

TABLE VIII.   EFFECTS OF VARIATIONS IN THE INITIAL
PARAMETER VARIANCES ON NEAR STEADY-
STATE PARAMETERS AND THEIR VARIANCES
(LINEAR SYSTEM, MEASUREMENT ERRORS
AND SYSTEM NOISE)

Variation in Initial Parameter Variance* (Percent)	Errors in Near Steady-State Parameters (Percent)		Normalized Near Steady-State Variances (Percent)	
	$\hat{X}_3$	$\hat{X}_4$	$\dfrac{\sqrt{P_{33}}}{n}$	$\dfrac{\sqrt{P_{44}}}{n}$
-25	0.72	-0.47	2.42	14.39
0	0.88	2.66	2.42	15.18
25	0.98	4.95	2.42	15.81
*relative to sample variance				

than one standard deviation of their estimated
value regardless of the initial parameter variance,
when the standard deviation is taken as the square
root of the near steady-state parameter variance.
Attention is also called to the fact that since
system noise is now present in the data, the near
steady-state parameter variances are of greater
magnitude than was the case when only measurement
errors were present (see Figures 24, 25, 31, and
32).

(3) Filter Response to Large Errors in the Initial
Parameter Estimates

The results of using the extended Kalman filter to
identify the damping parameter X_4, when the initial
estimate of the parameter is in error by 100 percent
are shown in Figure 33 for two initial parameter
variance estimates. When the initial parameter
variance is based on the initial estimate of X_4, the
near steady-state parameter estimate is approximately
14 percent high; for an initial parameter variance
that is greater by approximately 25 percent, the
resulting near steady-state value of X_4 is nearly
19 percent high. The corresponding parameter variances
1.5 seconds after the filter begins integrating are
such that the estimated values of X_4 are within
approximately one standard deviation of the true value.

(4) Comparison of Filter Performance to Chapman-Kirk
Technique

The parameters of interest and their uncertainties
were extracted from the given noisy data set using
the previously described technique of Chapman and
Kirk in order to be compared to the filter results.
The percent error in the parameters and their normalized
estimated standard deviations obtained with the
Chapman-Kirk program are

X_3:12.5 percent (1.8 percent)

X_4: 3.1 percent (0.3 percent)

The above results are similar to those presented in
Table VI in that the estimated parameter uncertain-
ties are not of sufficient magnitude to reflect the
true error in the extracted parameters. Whereas the
extended filter estimates the parameters and variances

58

so that the true value of the parameter is generally within one standard deviation of the estimate, it is shown that such is not the case when using the Chapman-Kirk technique. The errors in the above parameters are also slightly greater than those obtained with the extended filter, although this is probably of less consequence than the differences in the uncertainties computed by the two techniques.

c. Nonlinear System with Measurement Errors Only

The pattern used in the previous two sections of generating a data set, investigating the basic response of the filter in identifying the parameters of interest, and checking the sensitivity of the filter to variations in the initial parameter variances is essentially repeated here for the more complicated system model

$$\ddot{\alpha} + (C_4 + C_6 \alpha^2)\dot{\alpha} + (C_3 + C_5 \alpha^2)\alpha = 0 \quad . \quad (88)$$

In addition, some effects of errors in the measurement error variance are also included and discussed. Some comparisons of results achieved with the extended filter and the Chapman-Kirk technique are also made.

The standard deviation of the measurement errors in the generated data is 0.00569 radian. The mean value of the errors is zero.

(1) Basic Filter Performance

The time histories of the four parameters of interest as computed by the extended filter while analyzing the data just described are shown in Figures 34, 35, 36 and 37. The initial errors in the parameters are -25 percent for X_3, X_4, and X_6 and -5 percent for X_5. The measurement error variance is that obtained from the simulation. The initial values of the parameter variances are based on the difference between the true values of the parameters and the above-mentioned initial estimates.

The reason the initial estimate of X_5 is only 5 percent low instead of 25 percent, as is the case with the other parameters, is due to its magnitude and a weakness in the numerical scheme. The true value of X_5 is 1960 based on the data in Table III. A 25 percent initial error in this parameter results in an initial parameter sample variance of

59

$$P_{55}(0) = (1960-(0.25)1960)^2 = 240,000$$

which is of sufficient magnitude to cause the filter
to diverge for the numerical integration step size
of 0.005 second. The divergence occurs because the
large initial value of P_{55} results in large values
of P_{25} and P_{15} which, in turn, cause the values of
$\dot{P}_{55}$ that are fed into the numerical integration sub-
routine to be very large [Equations (81) and (82)].
The result of this cascading effect is a rapid de-
crease in P_{55} in large increments until it becomes
negative (a physical impossibility since this term
is the variance of a parameter). The structure of
the covariance equations is such that a negative
variance causes numerical divergence in the solutions
to several of the equations.

To circumvent the problem, an initial error of
-10 percent in X_5, with a corresponding adjustment
in $P_{55}(0)$, was tried. The result was again diver-
gence. Finally, an initial estimate of -5 percent
was found to be successful.

The results shown in Figures 34, 35, and 36 indicate
that the filter does an excellent job in correctly
identifying the linear and nonlinear static restoring
moment parameters, X_3 and X_5, as well as the linear
pitch damping term, X_4. The results for the non-
linear damping parameter, X_6, are not as gratifying
as can be seen by referring to Figure 37. The error
in this term is still approximately -10 percent
after 1.5 seconds. The reason for this problem is
related to the magnitude of this parameter as com-
pared to others in the equation of motion. The true
value of X_3 and X_5 are 160 and 1960, respectively,
although the magnitude of X_5 is effectively reduced
by at least an order of magnitude when it is multiplied
by α^3. X_4 and X_6, on the other hand, have true values
of 1.60 and 4.35, respectively. After X_6 is multiplied
by $\alpha^2\dot{\alpha}$, it is effectively the smallest parameter in
the equation of motion by an order of magnitude and
it is naturally more difficult to identify since it
has the least effect on the trajectory.

(.) Effects of Variations in the Initial Parameter
Variances

The effects of variations in the initial parameter
variances are summarized in Table IX and in Figures
38 and 39. As can be seen from these results, the

60

TABLE IX. EFFECTS OF VARIATIONS IN THE INITIAL PARAMETER VARIANCES ON NEAR STEADY-STATE PARAMETERS AND THEIR VARIANCES (NON-LINEAR SYSTEM, MEASUREMENT ERRORS ONLY)

Variation in Initial Parameter Variance* (Percent)	Errors in Near Steady-State Parameters (Percent)				Normalized Near Steady-State Variances (Percent)			
	$\hat{X}_3$	$\hat{X}_4$	$\hat{X}_5$	$\hat{X}_6$	$\sqrt{P_{33}}_n$	$\sqrt{P_{44}}_n$	$\sqrt{P_{55}}_n$	$\sqrt{P_{66}}_n$
-50	-0.04	1.18	-0.10	-13.4	0.44	1.15	0.35	10.4
-25	-0.10	1.02	-0.01	-12.1	0.44	1.22	0.36	10.9
0	-0.13	0.94	0.04	-11.6	0.44	1.23	0.34	10.7
25	-0.15	0.93	0.06	-11.7	0.39	1.17	0.15	8.9

* relative to sample variance

61

near steady-state values of the parameters and their variances are relatively insensitive over the range of initial parameter variances considered. Cnly the X_6 parameter estimate and its variance show a variation of more than 1 percent. As is obvious from the referenced table and figures, no data are available for initial parameter variances of 50 percent above the sample variances; the reason for this is, again, that the magnitude of the P_{55} variance caused the filter to diverge.

Attention is called to the fact that, for the range of initial variances considered, the near steady-state estimates of the X_3, X_4, and X_5 parameters are all within one standard deviation of their true values when the standard deviation is taken as the square root of the near steady-state variance for the parameter of interest. The X_6 parameter is always within two standard deviations of its true value.

(3) Effects of Errors in the Estimate of the Measurement Error Variance

Up to this point, exact values of the measurement error variance, r, have been used in all runs made with the extended filter. Exact knowledge of this quantity is available because the pitch-angle data being used with the extended filter are computer-generated with specified noise statistics. In actual test situations, true values of the noise parameters may not be known exactly and, in fact, methods for determining both measurement error variances and system noise variances from noisy dynamic data are of current research interest.[34]

Some effects of errors in the estimates of the measurement error variance on the near steady-state parameter estimates and their uncertainties are given in Table X and in Figures 40 and 41. These results show that errors in the measurement noise variance of ± 25 percent have essentially no effect on estimates of the four parameters of interest. The uncertainties of the X_3, X_4, and X_5 parameters are changed by less than 1 percent over the range considered. The normalized X_6 uncertainty varies from approximately 5 percent to 12 percent.

TABLE X. EFFECTS OF ERRORS IN THE ESTIMATE OF THE MEASUREMENT ERROR VARIANCE ON NEAR STEADY-STATE PARAMETERS AND THEIR VARIANCES (NONLINEAR SYSTEM, MEASUREMENT ERRORS ONLY)

Error in Estimate of Measurement Error Variance, r (Percent)	Errors in Near Steady-State Parameters (Percent)				Normalized Near Steady-State Variances (Percent)			
	$\hat{X}_3$	$\hat{X}_4$	$\hat{X}_5$	$\hat{X}_6$	$\sqrt{P_{33}}\ n$	$\sqrt{P_{44}}\ n$	$\sqrt{P_{55}}\ n$	$\sqrt{P_{66}}\ n$
-25	-0.14	0.95	0.05	-12.2	0.27	0.91	0.0	4.93
0	-0.13	0.94	0.04	-11.6	0.44	1.23	0.34	10.73
25	-0.10	1.00	0.00	-11.9	0.50	1.37	0.40	12.19

(4) Filter Response to Larger Error in the Initial Estimate of X_5

The difficulty associated with numerically large initial parameter variance estimates has been discussed earlier. The sample variance of P_{55} which corresponds to a -5 percent error in the initial X_5 parameter estimate appears to be an approximate upper limit for the integration step size and physical constants used in this analysis. A run was made, however, with the previously described nonlinear data set containing only measurement errors where all initial parameter estimates were in error by -25 percent; the P_{55} variance, however, was set equal to the previously used sample variance which corresponds to a -5 percent initial estimate of X_5. The parameter estimate time histories resulting from this initialization are shown in Figures 42 through 45. These results indicate that the near steady-state values of the parameters are very nearly equal to those presented when the sample variances based on the initial parameter estimates were used initially.

(5) Comparison of Filter Performance to Chapman-Kirk Technique

The linear and nonlinear static restoring moment and damping parameters were extracted from the noisy data set being considered in this section using the Chapman-Kirk program. The percent errors in the parameters and their normalized estimated uncertainties are given below.

X_3:0.36 percent (0.35 percent)

X_4:0.69 percent (0.43 percent)

X_5:0.41 percent (0.49 percent)

X_6:4.60 percent (11.27 percent)

Comparing the above results with those previously given in Table IX reveals that the estimates obtained using the extended filter are slightly more accurate for all but the X_4 parameter. Since the errors are, for the most part, less than 1 percent, the differences are essentially insignificant. The normalized uncertainties obtained with both techniques are equally good.

64

d. **Nonlinear System with Measurement Errors and System Noise**

In this section, as before, the basic filter performance, sensitivity of the filter to initial parameter variance variations and sensitivity of the filter to errors in the estimates of the noise variances are investigated. The data used in the analysis were generated by numerically integrating the following nonlinear equation with a random forcing function,

$$\ddot{\alpha} + (C_4 + C_6 \alpha^2)\dot{\alpha} + (C_3 + C_5 \alpha^2)\alpha = w(t) \quad . \quad (89)$$

Approximately the same standard deviations used previously are repeated once again; the standard deviation of the measurement errors is 0.00576 radian and the standard deviation of the system noise is 4.75 rad/sec^2.

(1) **Basic Filter Performance**

The time histories of the errors in the parameters of interest obtained with the extended filter are given in Figures 46 through 49. The response time required for the parameters to reach their near steady-state values is slightly greater than in the simpler cases that have been considered previously. The accuracy of the parameters is similar to that which was achieved for the nonlinear system with only measurement errors in the data with the exception of X_4, which is approximately 5 percent higher.

(2) **Effects of Variations in the Initial Parameter Variances**

The effects of variations in the initial parameter variances are given in Table XI and are shown in Figures 50 and 51. Only the nonlinear damping parameter and its uncertainty vary by more than 1 percent for initial variance variations of up to 50 percent relative to the sample variances.

(3) **Effects of Errors in the Estimates of the Noise Variances**

The effects of errors in the estimates of r, the measurement error variance, and q, the system noise variance, are given in Table XII and are shown in

65

Figures 52 and 53. These figures show the
variation of the near steady-state parameter
errors and normalized parameter uncertainties
as functions of the error in the estimate of the
measurement error variance for various values of
error in the system noise variance estimate. From
these results it is obvious that only X_6 is
affected by errors of ± 25 percent in the estimate
of the system noise variance or measurement error
variance.

(4) Comparison of Filter Performance to Chapman-Kirk
Technique

The percent errors in the parameters and their
normalized estimated standard deviations that are
obtained using the Chapman-Kirk program with the
data currently being analyzed are given below:

X_3:0.62 percent (0.40 percent)

X_4:2.63 percent (1.23 percent)

X_5:0.41 percent (0.61 percent)

X_6:46.3 percent (14.8 percent)

Comparing these results with the results in Table
XI indicates that both techniques yield essentially
the same accuracy in their determination of X_3 and
X_5. The filter does considerably better in estimating
X_6 but is slightly worse in its estimate of X_4.

The uncertainties computed using the filter are of
sufficient magnitude so that the true value of the
X_3, X_5, and X_6 parameters are within one standard
deviation of the filter estimates; the true value
of X_4 is within two standard deviations.

When using the Chapman-Kirk technique, only X_5 is
within one estimated standard deviation of its
true value; X_3 and X_4 are within two estimated
standard deviations of their respective true values.
The extracted value of X_6 is not within three estimated
standard deviations of its true value.

66

TABLE XI. EFFECTS OF VARIATIONS IN THE INITIAL PARAMETER VARIANCES ON NEAR STEADY-STATE PARAMETERS AND THEIR VARIANCES (NON-LINEAR SYSTEM, MEASUREMENT ERRORS AND SYSTEM NOISE)

Variation in Initial Parameter Variance* (Percent)	Errors in Near Steady-State Parameters (Percent)				Normalized Near Steady-State Variances (Percent)			
	$\hat{X}_3$	$\hat{X}_4$	$\hat{X}_5$	$\hat{X}_6$	$\sqrt{P_{33}}\,n$	$\sqrt{P_{44}}\,n$	$\sqrt{P_{55}}\,n$	$\sqrt{P_{66}}\,n$
-50	-0.36	6.38	0.71	-16.9	1.74	4.27	1.16	16.7
-25	-0.57	6.51	0.87	-14.0	1.76	4.52	1.18	20.3
0	-0.59	6.40	0.92	-11.4	1.77	4.71	1.19	23.2
25	-0.58	6.17	0.92	-8.8	1.77	4.87	1.17	25.5

*relative to sample variance

67

TABLE 4.1.1. EFFECTS OF ERRORS IN THE ESTIMATES OF NOISE VARIANCES ON NEAR STEADY-STATE PARAMETERS AND THEIR VARIANCES (NONLINEAR SYSTEM, MEASUREMENT ERRORS AND SYSTEM NOISE)

Errors in Estimates of Noise Variances (Percent)		Errors in Near Steady-State Parameters (Percent)				Normalized Near-Steady-State Variances (Percent)			
q	r	$\hat{x}_3$	$\hat{x}_4$	$\hat{x}_5$	$\hat{x}_6$	$\sqrt{P_{33}}\,n$	$\sqrt{P_{44}}\,n$	$\sqrt{P_{55}}\,n$	$\sqrt{P_{66}}\,n$
-25	-25	-0.56	6.08	.90	-7.8	1.52	4.26	1.01	22.6
-25	0	-0.67	6.18	0.98	-8.4	1.57	4.29	1.07	22.9
0	-25	-0.5	6.38	0.88	-11.4	1.72	4.69	1.13	23.0
0	0	-0.59	6.40	0.92	-11.4	1.80	4.71	1.19	23.2
0	25	-0.62	6.43	0.95	-11.6	1.80	4.72	1.22	23.3
25	0	-0.52	6.51	0.87	-13.5	1.94	5.08	1.29	23.3
25	25	-0.54	6.51	0.89	-13.5	1.97	5.10	1.33	23.4
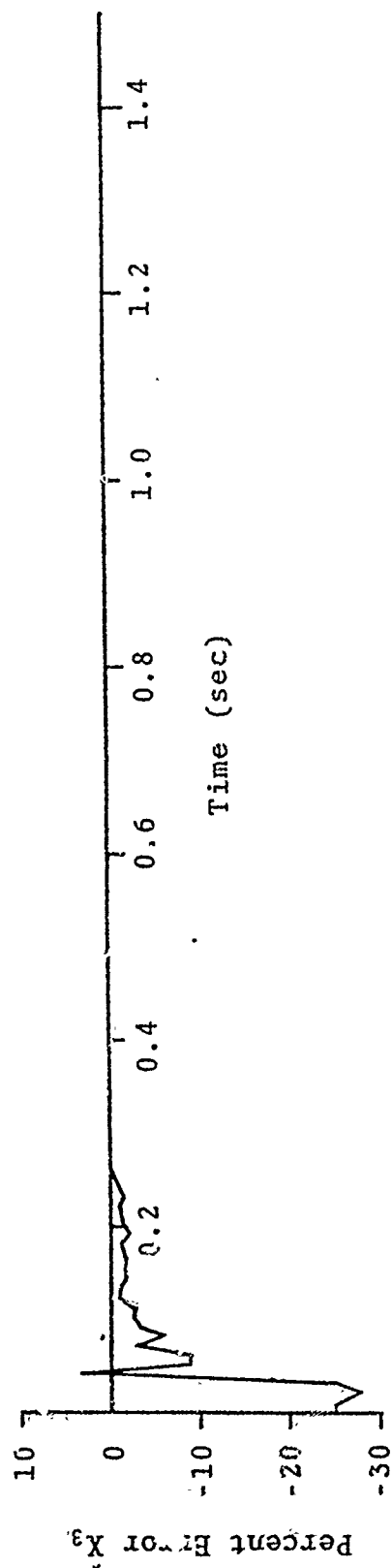
Figure 20. Variation of Percent Error in $\hat{X}_3$ with Time (Linear System, Measurement Errors Only)
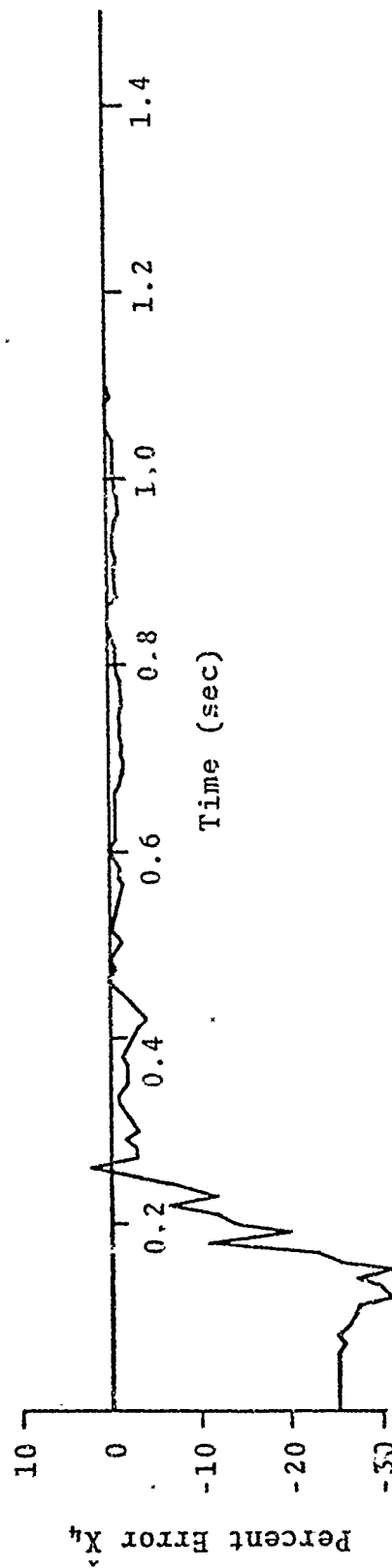
Figure 21. Variation of Percent Error in $\hat{X}_4$ with Time (Linear System, Measurement Errors Only)

69
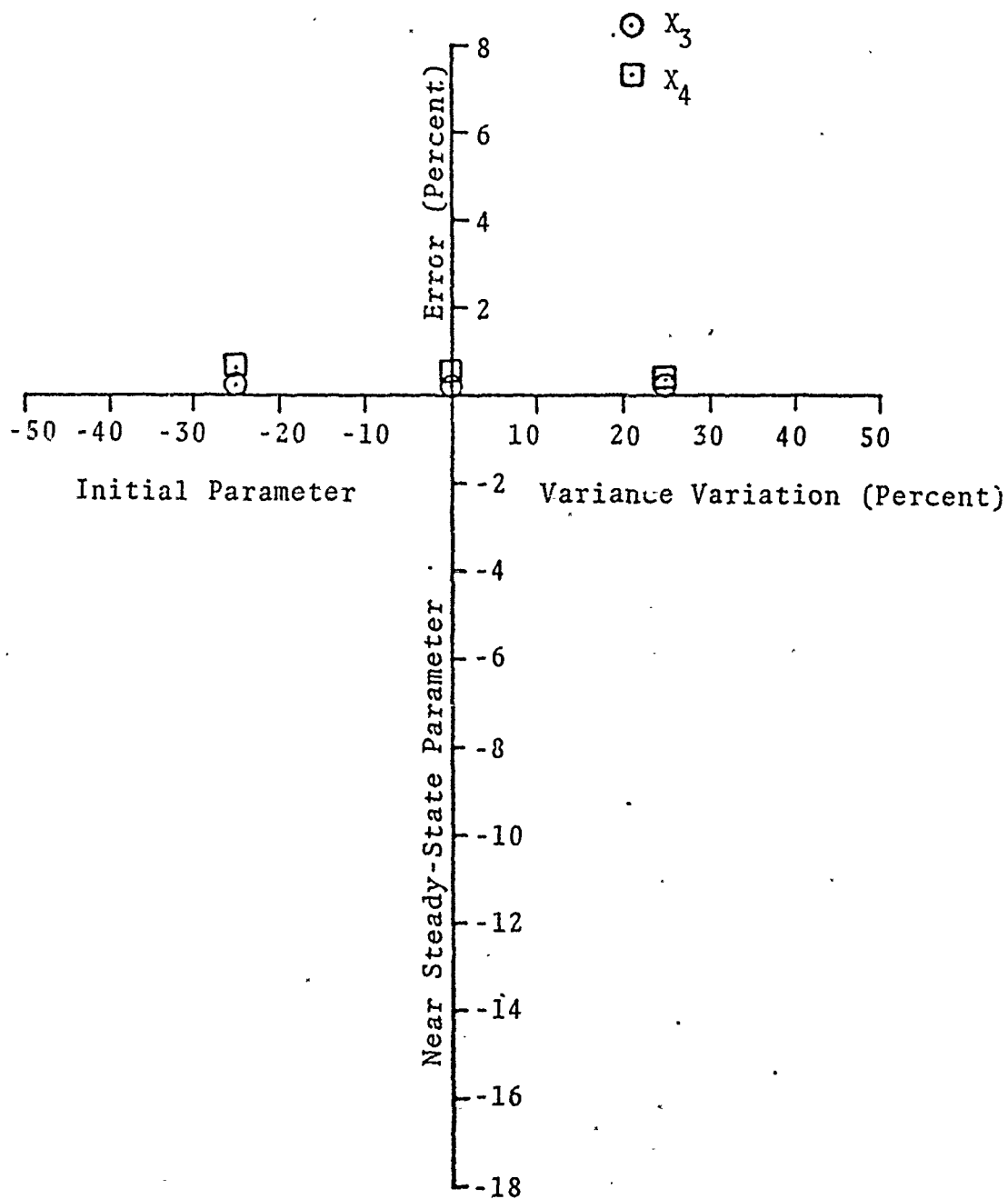
Figure 22. Variation of Near Steady-State Parameter
Error with Initial Parameter Variance
(Linear System, Measurement Errors Only)
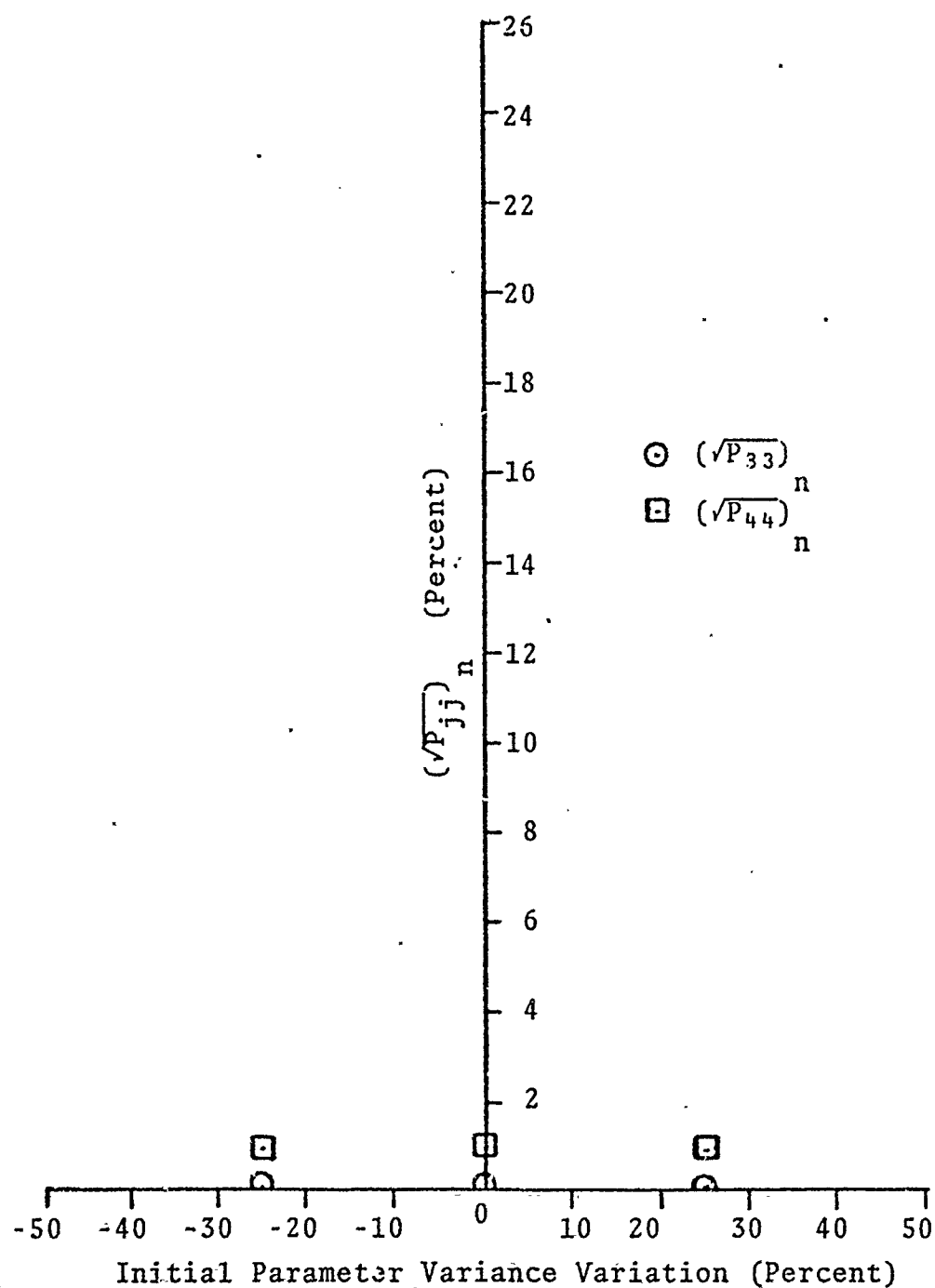
Figure 23. Variation of Normalized Near Steady-State
            Parameter Uncertainty with Initial Parameter
            Variance (Linear System Measurement Errors Only)

71
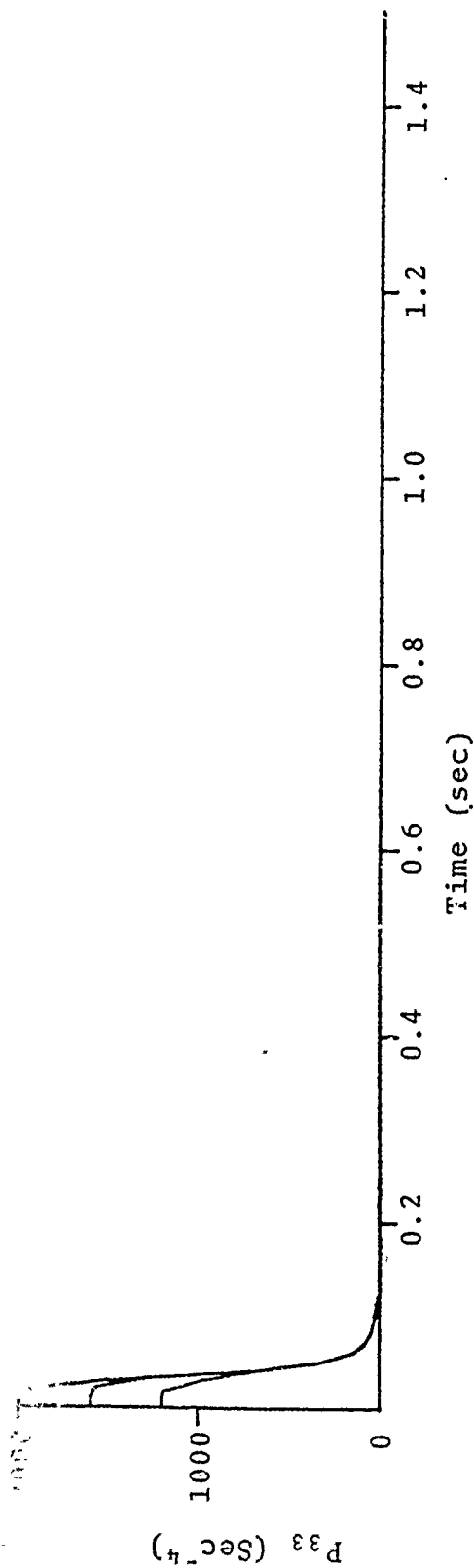
Figure 24. Variation of $\hat{X}_3$ Variance with Time for Three Initial Estimates (Linear System, Measurement Errors Only)
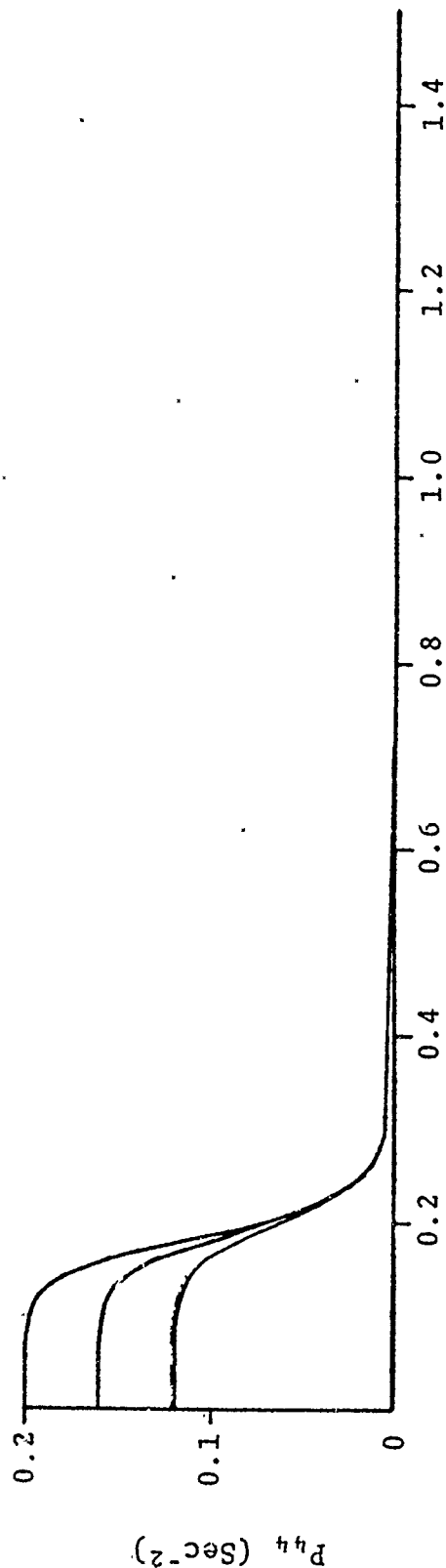
Figure 25. Variation of $\hat{X}_4$ Variance with Time for Three Initial Estimates (Linear System, Measurement Errors Only)

72
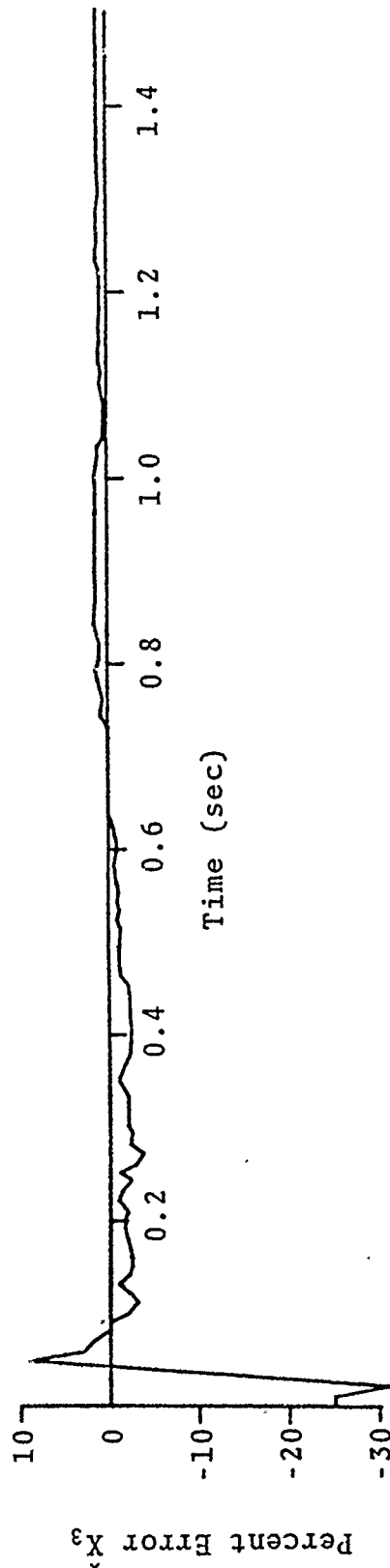
Figure 26. Variation of Percent Error in $\hat{X}_3$ with Time (Linear System, Measurement Errors and System Noise)
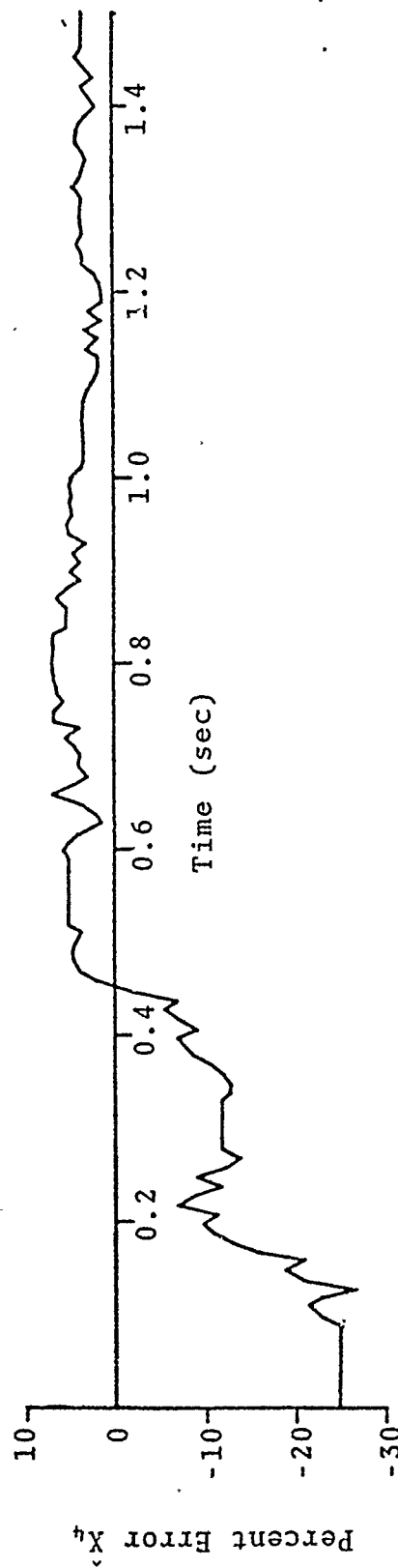
Figure 27. Variation of Percent Error in $\hat{X}_4$ with Time (Linear System, Measurement Errors and System Noise)

73
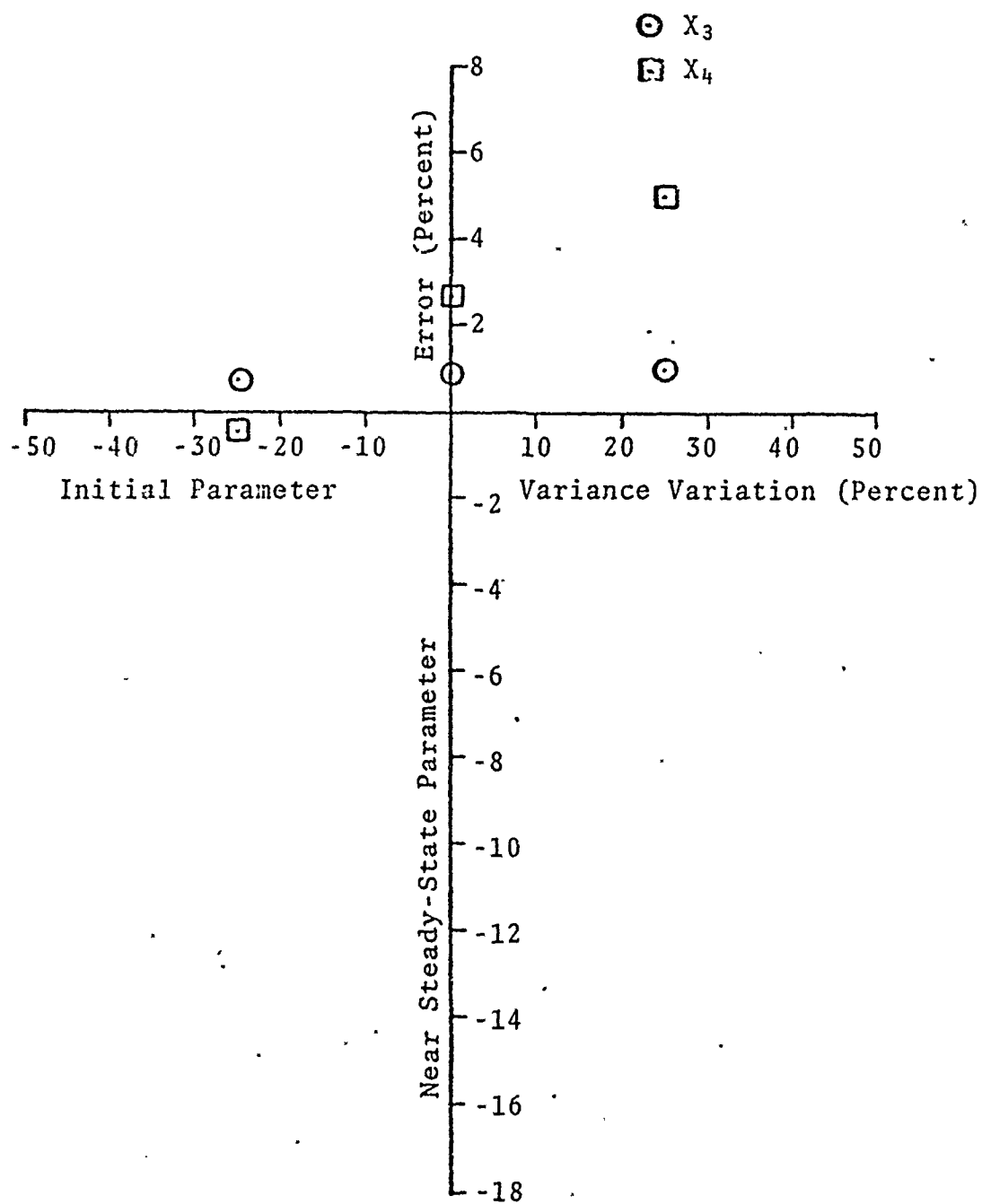
Figure 28. Variation of Near Steady-State Parameter
Error with Initial Parameter Variance
(Linear System, Measurement Errors and
System Noise)

74
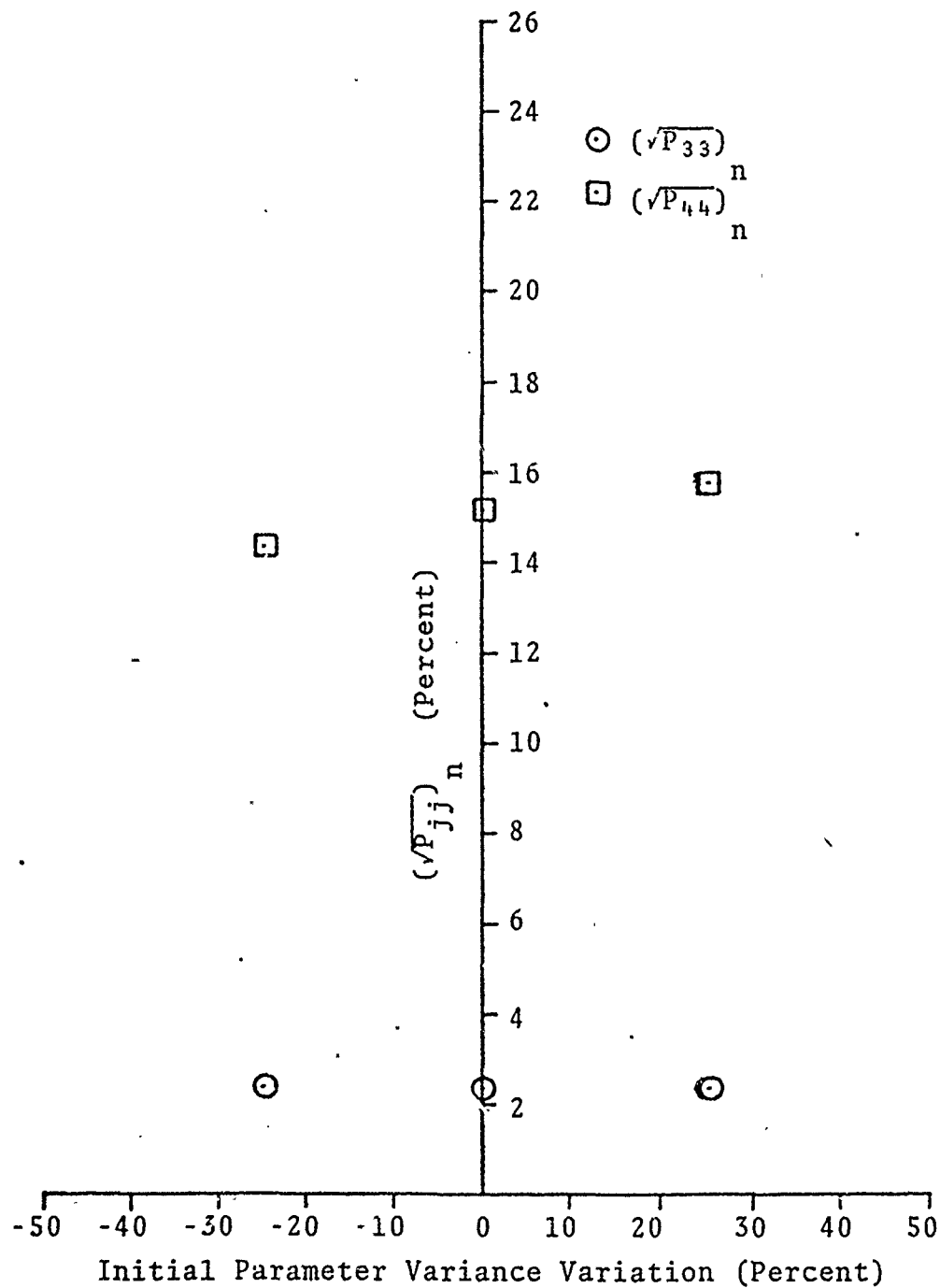
Figure 29. Variation of Normalized Near Steady-State Parameter Uncertainty with Initial Parameter Variance (Linear System, Measurement Errors and System Noise)
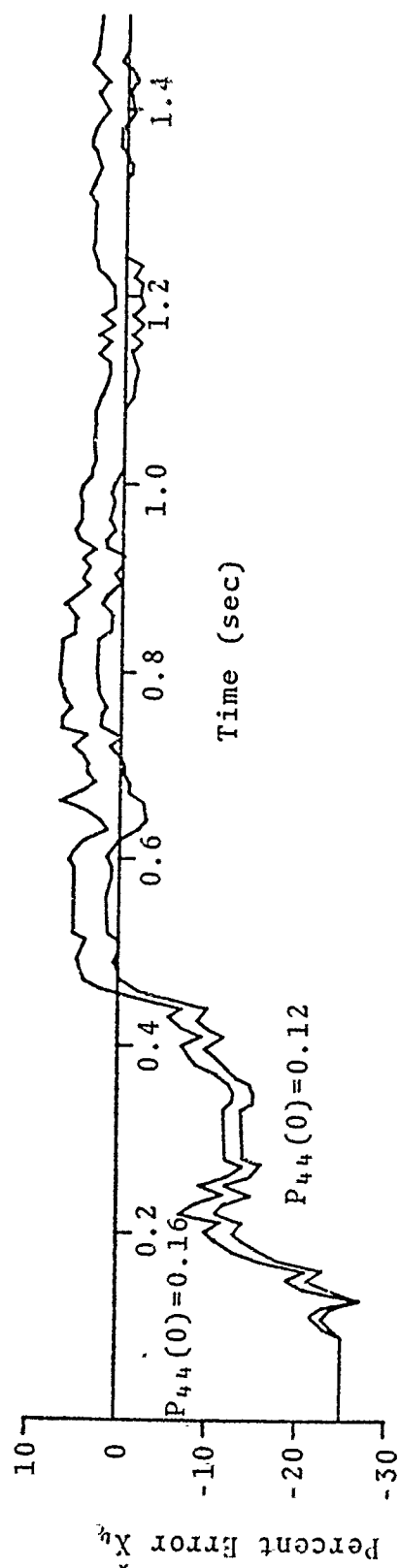
Figure 30. Variation of Percent Error in $\hat{X}_4$ with Time for Two Estimates of the Initial Parameter Variance (Linear System, Measurement Errors and System Noise)
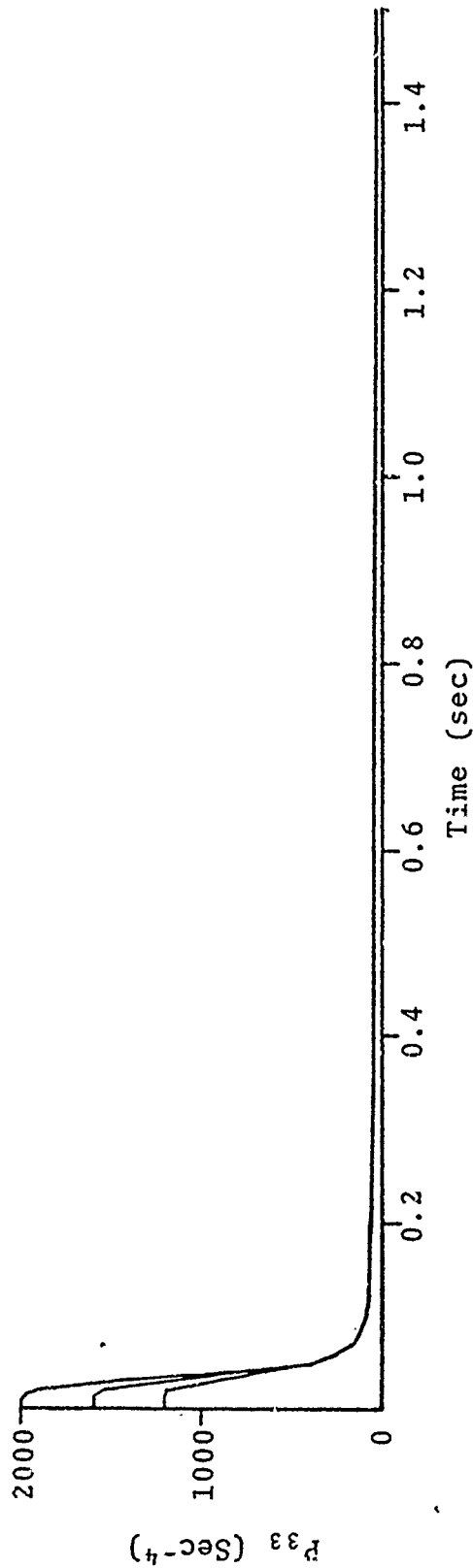
Figure 31. Variation of $\hat{X}_3$ Variance with Time for Three Initial Estimates (Linear System, Measurement Errors and System Noise)
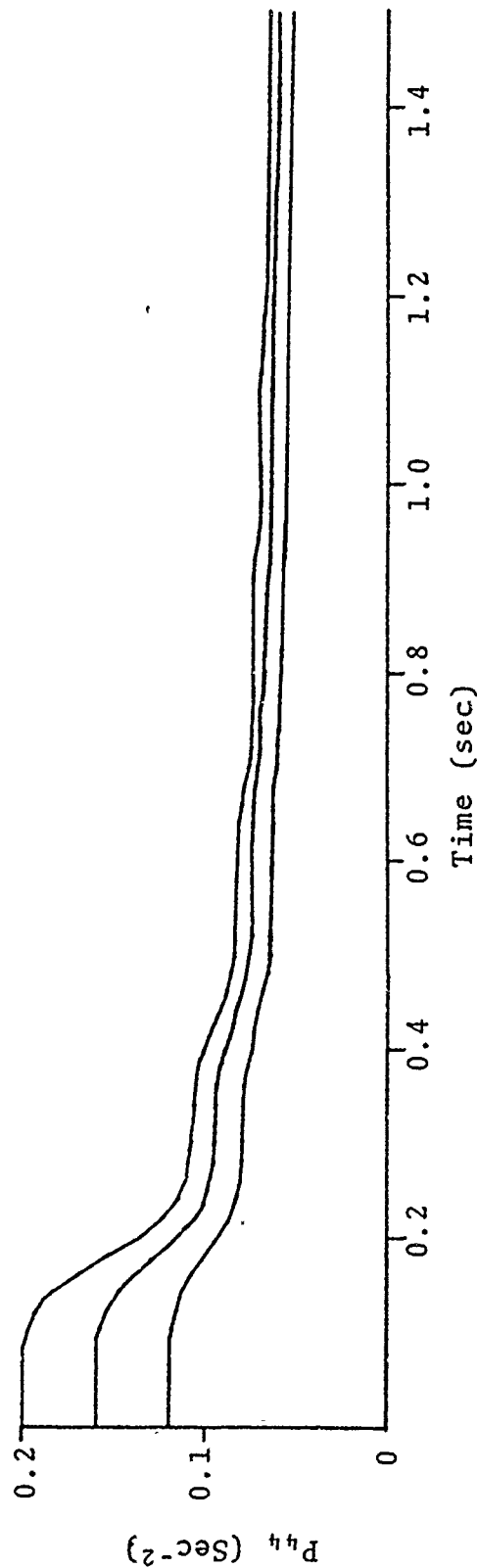
Figure 32. Variation of $\hat{X}_4$ Variance with Time for Three Initial Estimates (Linear System, Measurement Errors and System Noise)

77
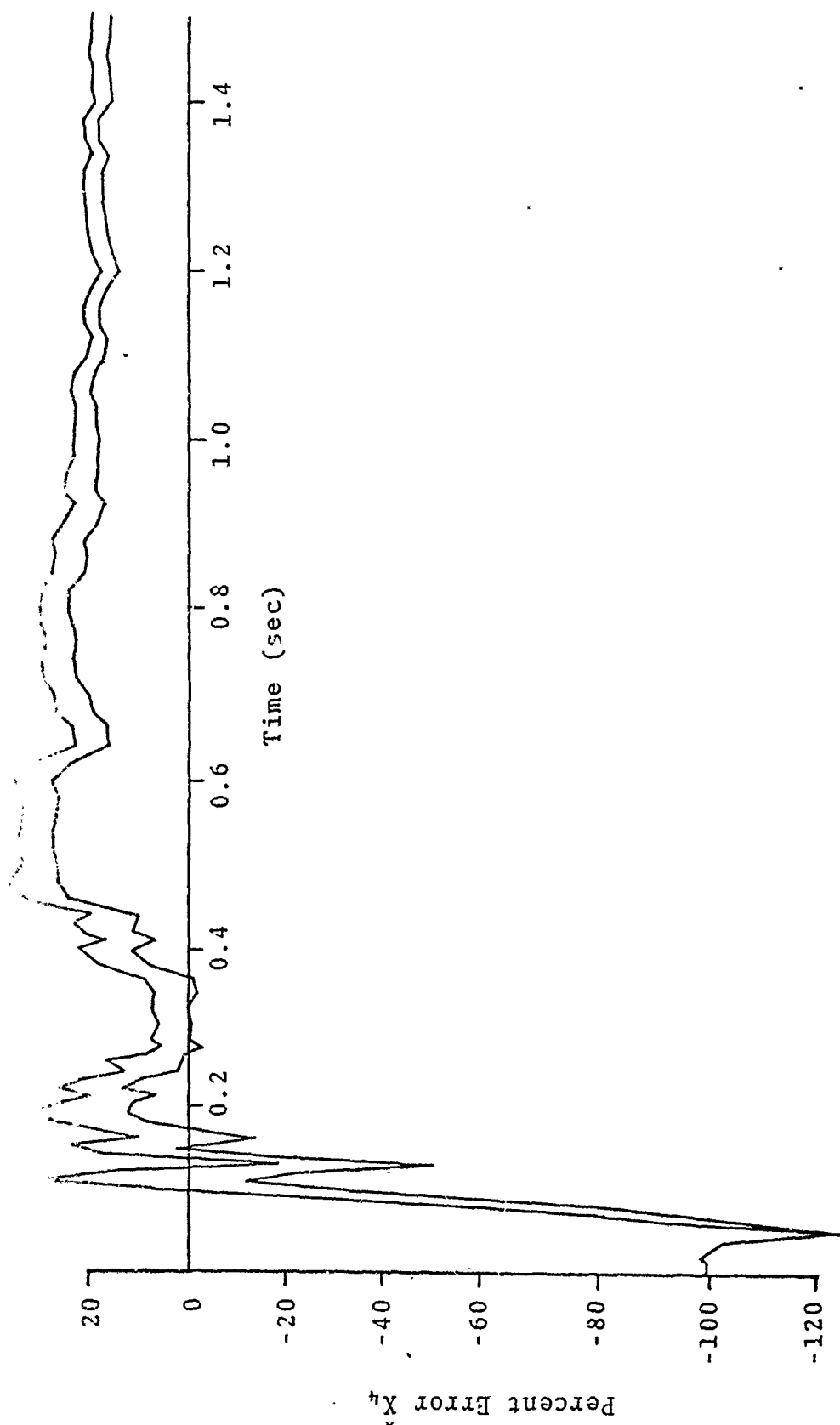
Figure 33. Variation of Percent Error in $\hat{X}_4$ with Time for Large Initial Parameter Estimate Error and Two Initial Parameter Variance Estimates (Linear System, Measurement Errors and System Noise)
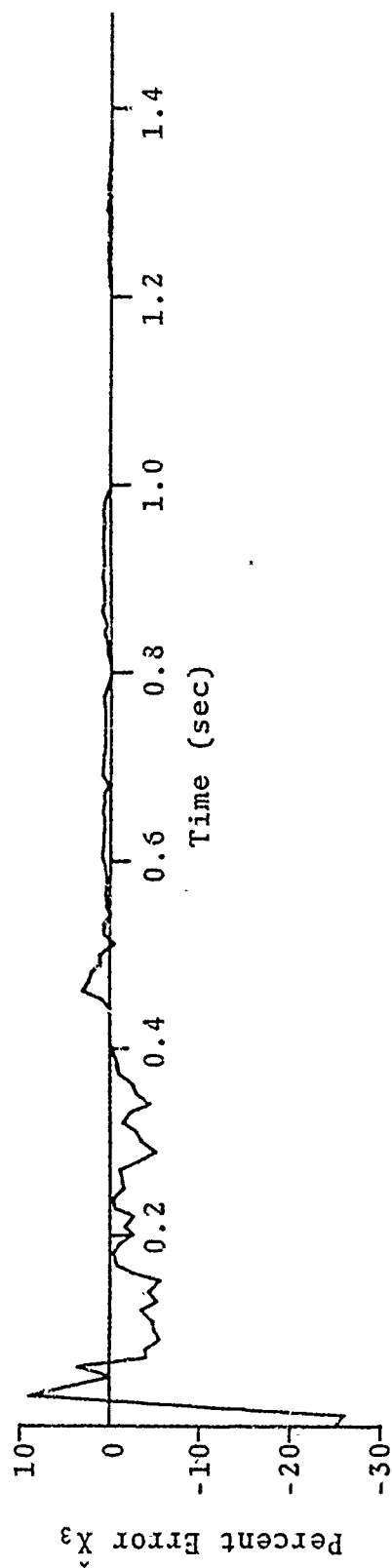
Figure 34. Variation of Percent Error in $\hat{X}_3$ with Time (Nonlinear System, Measurement Errors Only)
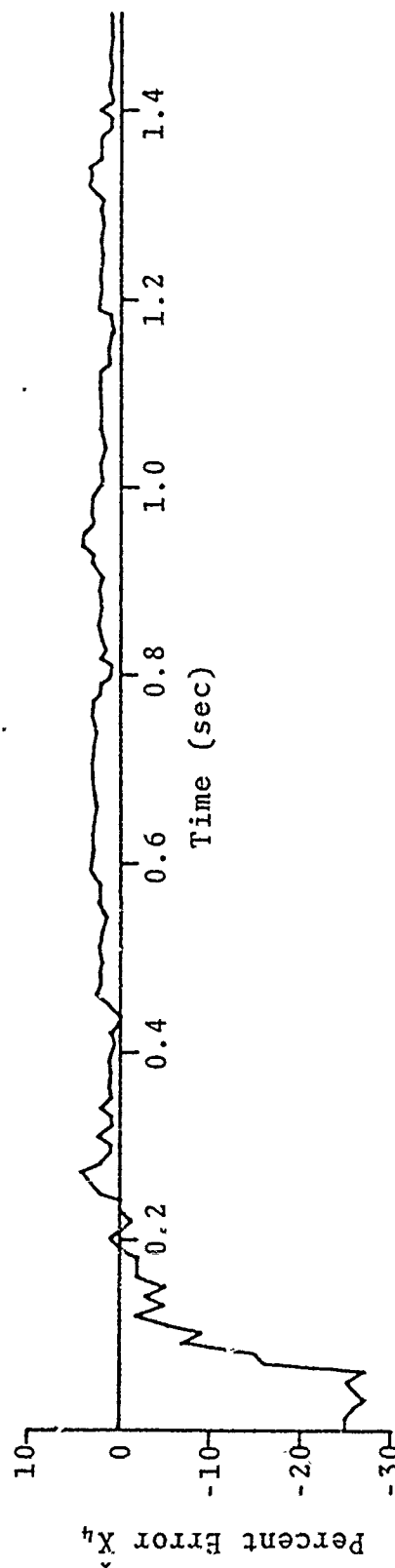
Figure 35. Variation of Percent Error in $\hat{X}_4$ with Time (Nonlinear System, Measurement Errors Only)

79
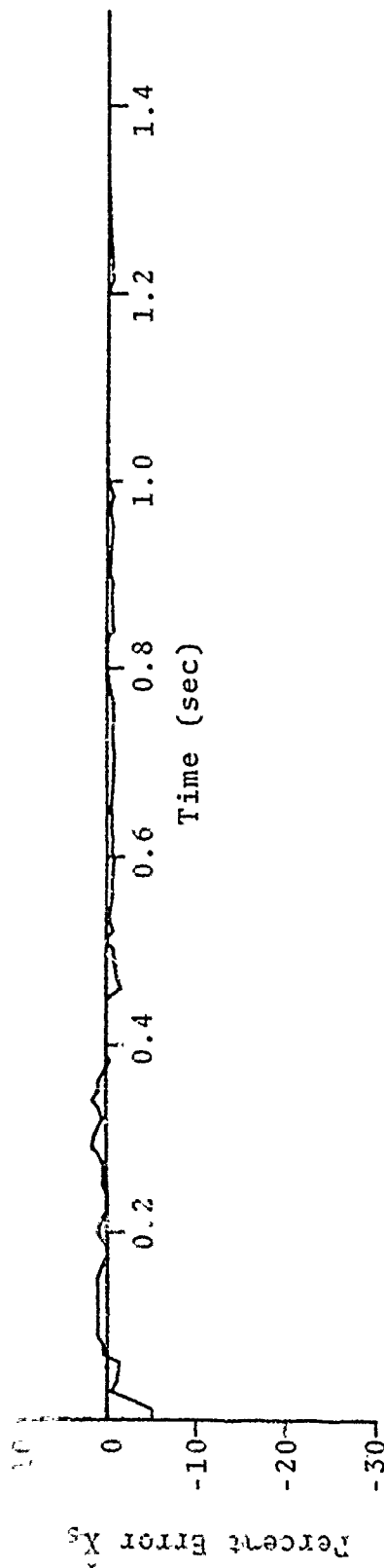
Figure 36. Variation of Percent Error in $\hat{X}_5$ with Time (Nonlinear System, Measurement Errors Only)
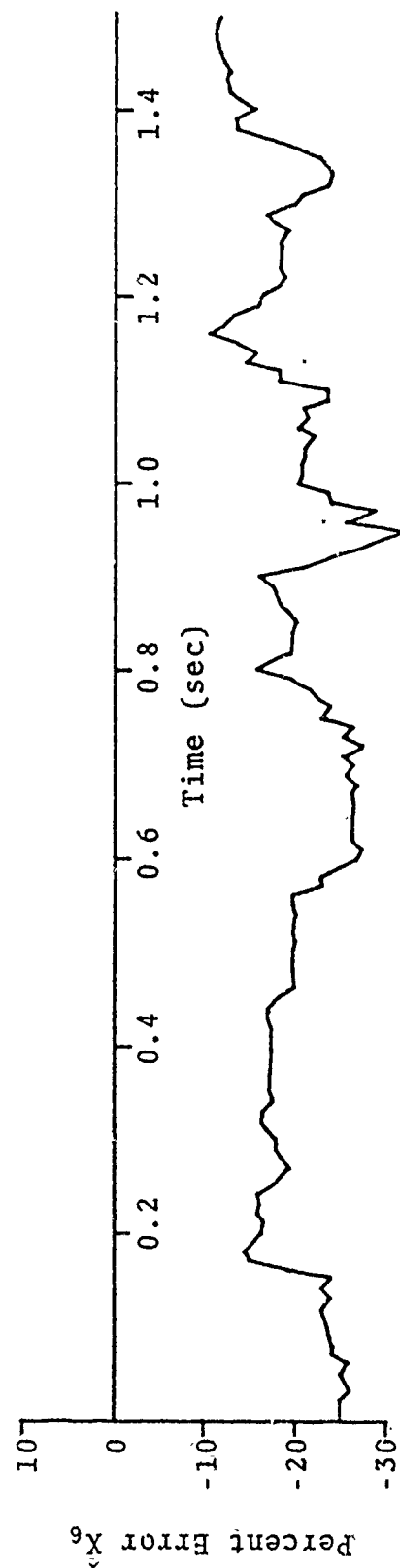
Figure 37. Variation of Percent Error in $\hat{X}_6$ with Time (Nonlinear System, Measurement Errors Only)

80
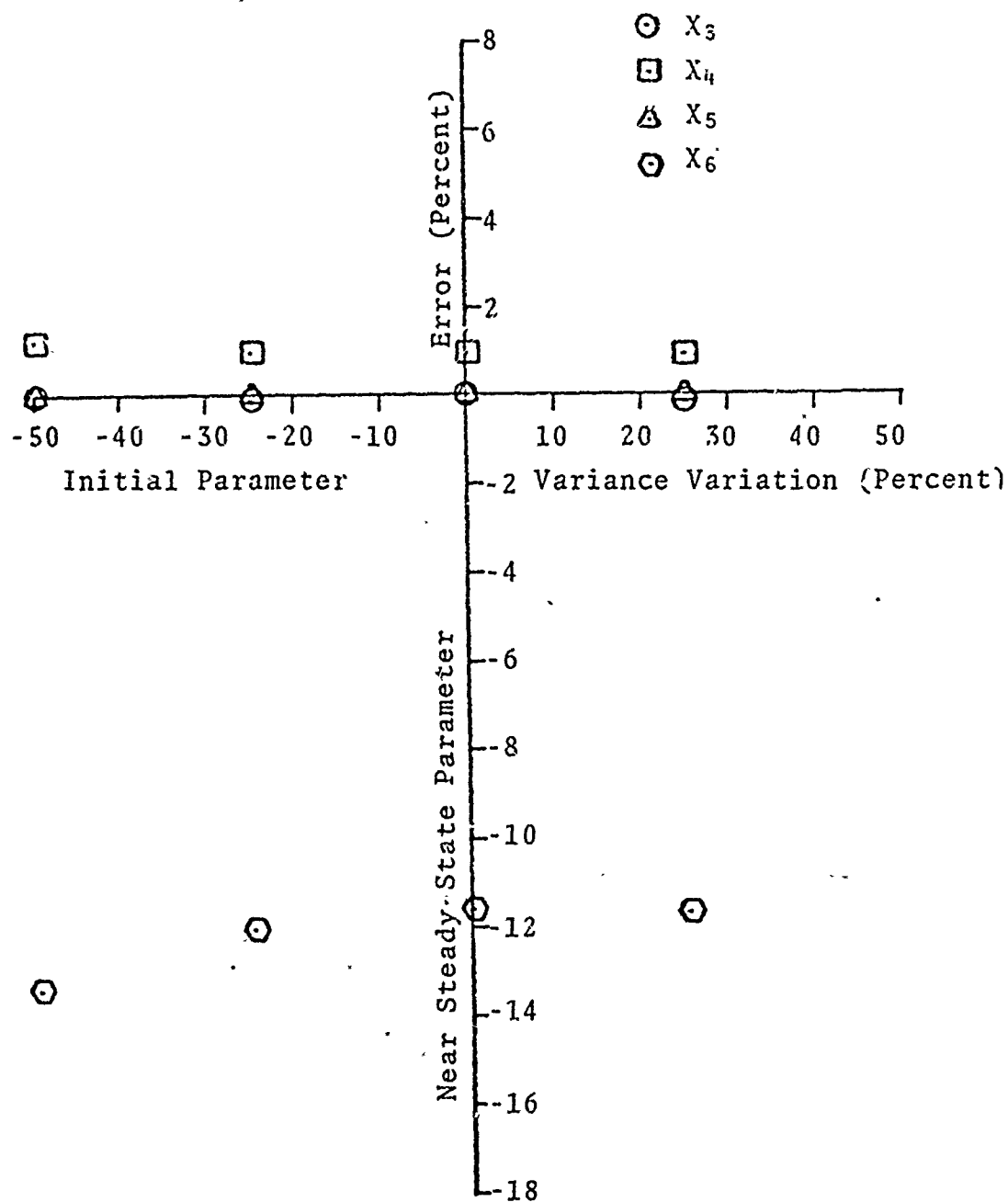
Figure 38. Variation of Near Steady-State Parameter Error
with Initial Parameter Variance (Nonlinear
System, Measurement Errors Only)

81
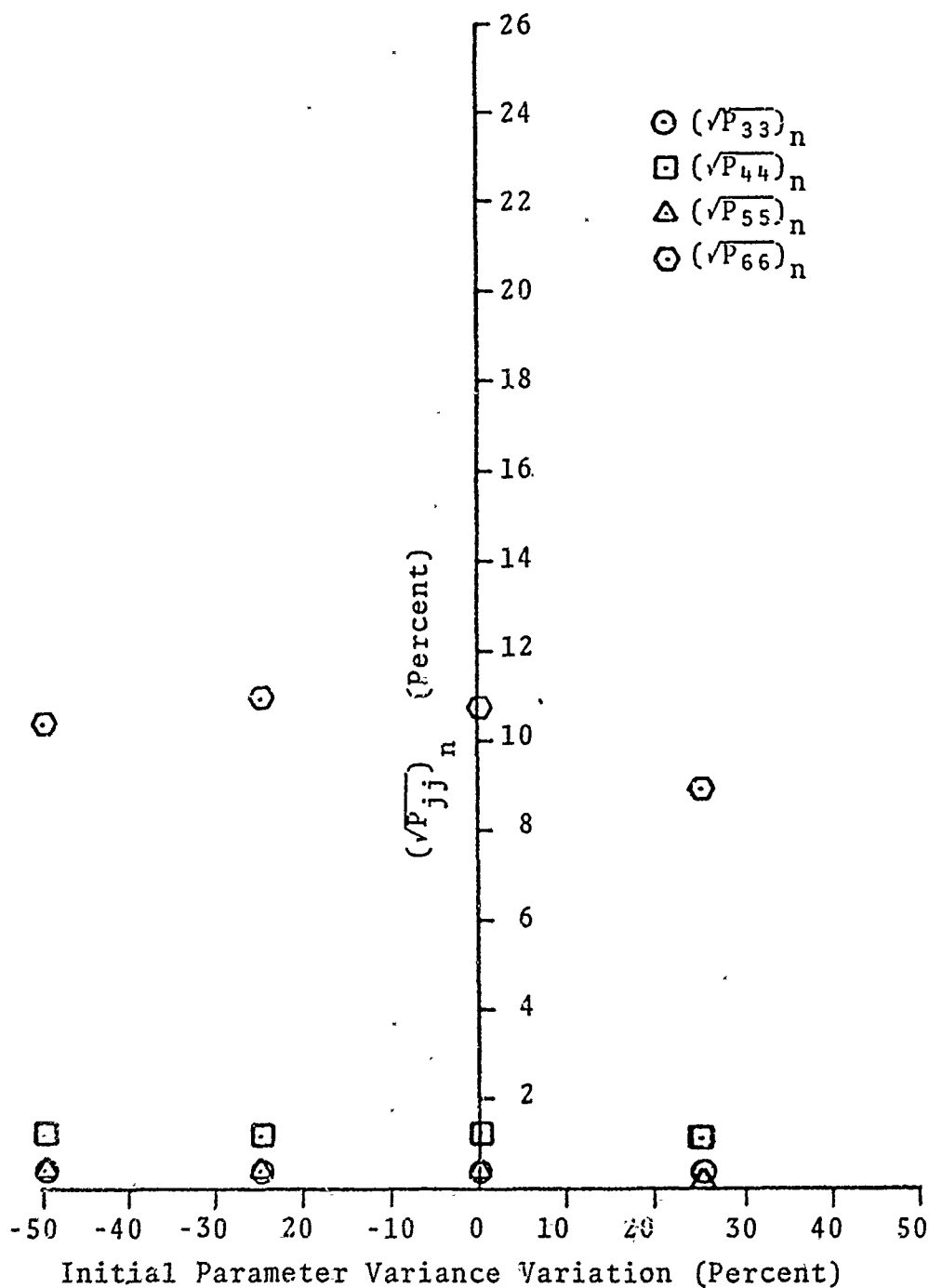
Figure 39. Variation of Normalized Near Steady-State Parameter Uncertainty with Initial Parameter Variance (Nonlinear System, Measurement Errors Only)

82
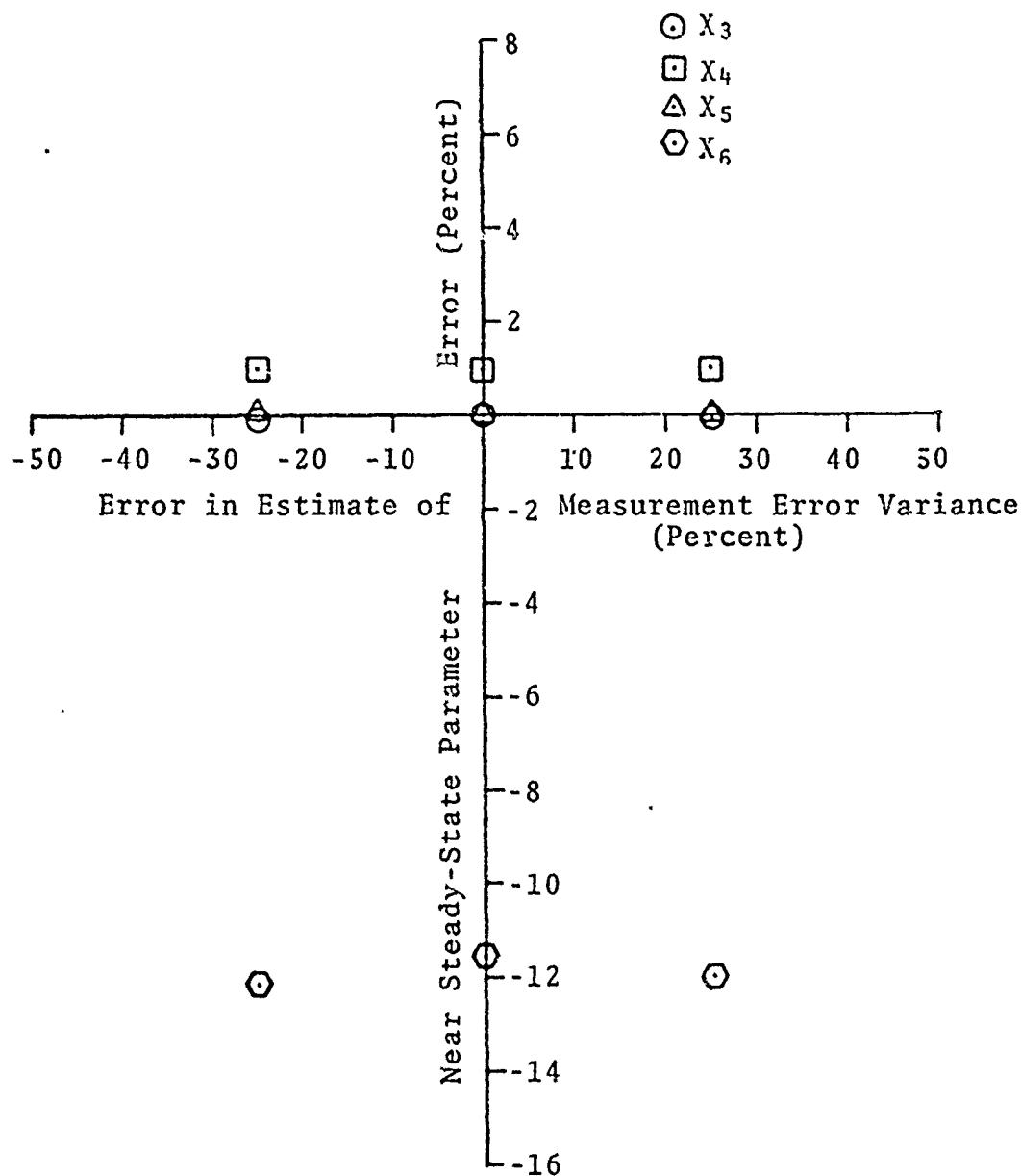
Figure 40.   Variation of Near Steady-State Parameter
Error with Error in the Estimate of the
Measurement Error Variance (Nonlinear
System, Measurement Errors Only)
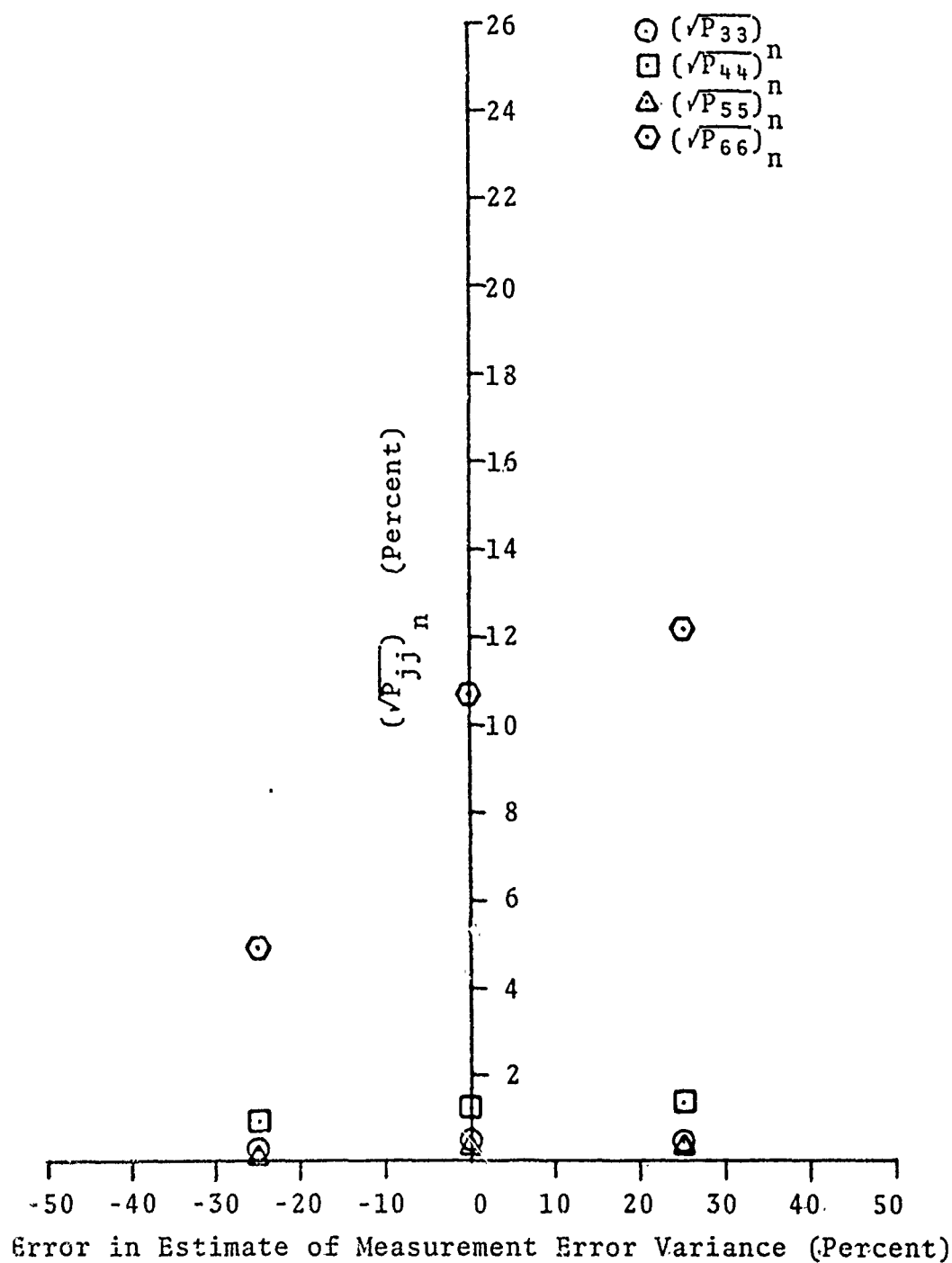
Figure 41. Variation of Normalized Near Steady-State Parameter Uncertainty with Error in the Estimate of Measurement Error Variance (Nonlinear System, Measurement Errors Only)
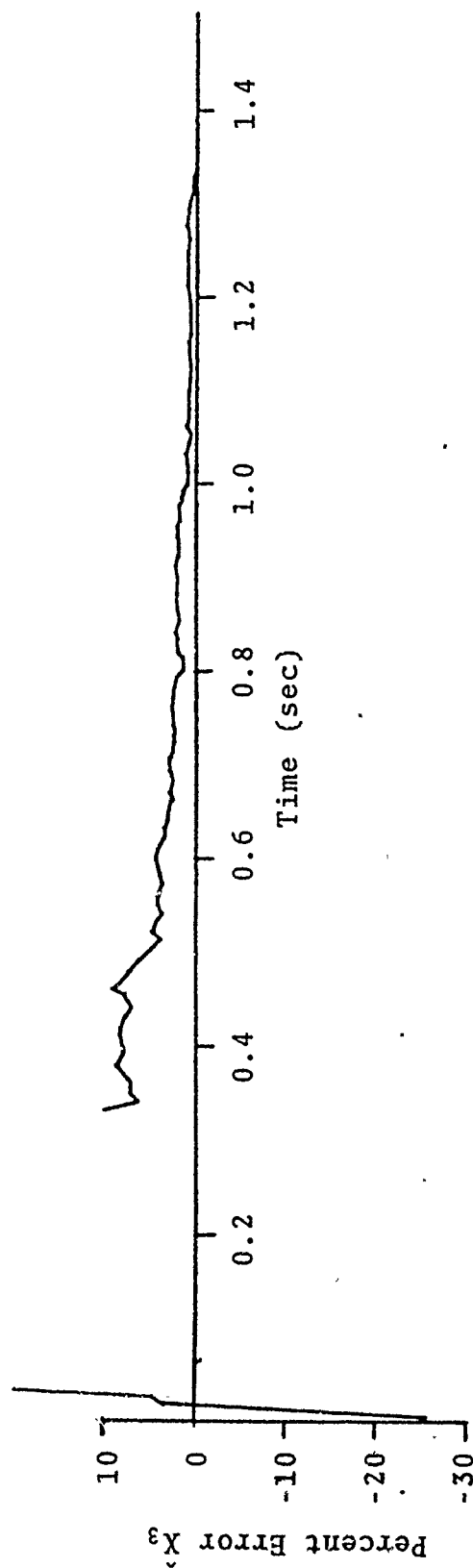
Figure 42. Variation of Percent Error in $\hat{X}_3$ with Time for Large Error in Initial Estimate of X_5 (Nonlinear System, Measurement Errors Only)
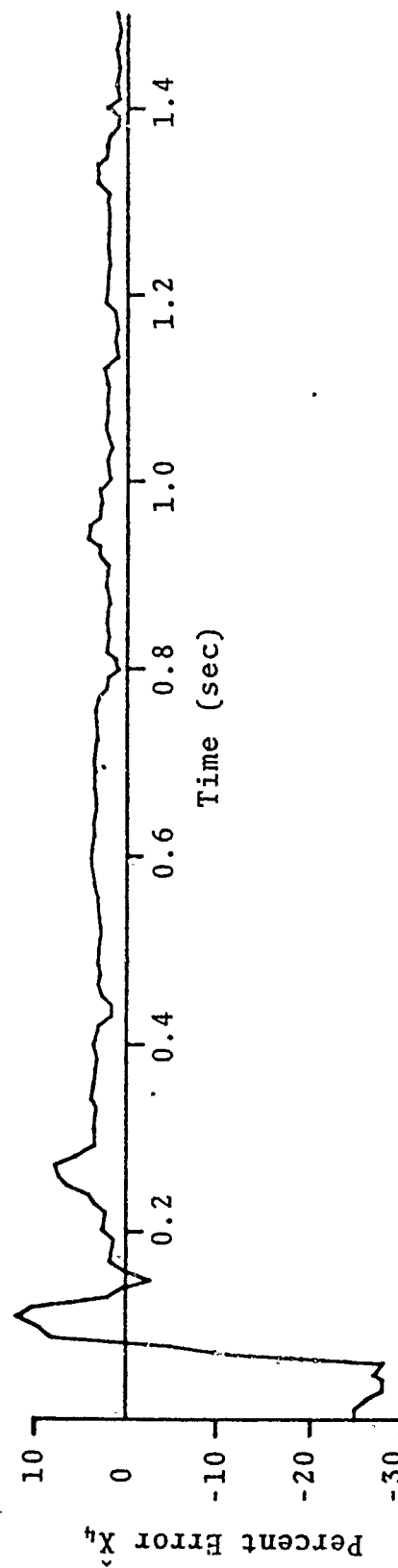
Figure 43. Variation of Percent Error in $\hat{X}_4$ with Time for Large Error in Initial Estimate of X_5 (Nonlinear System, Measurement Errors Only)

85
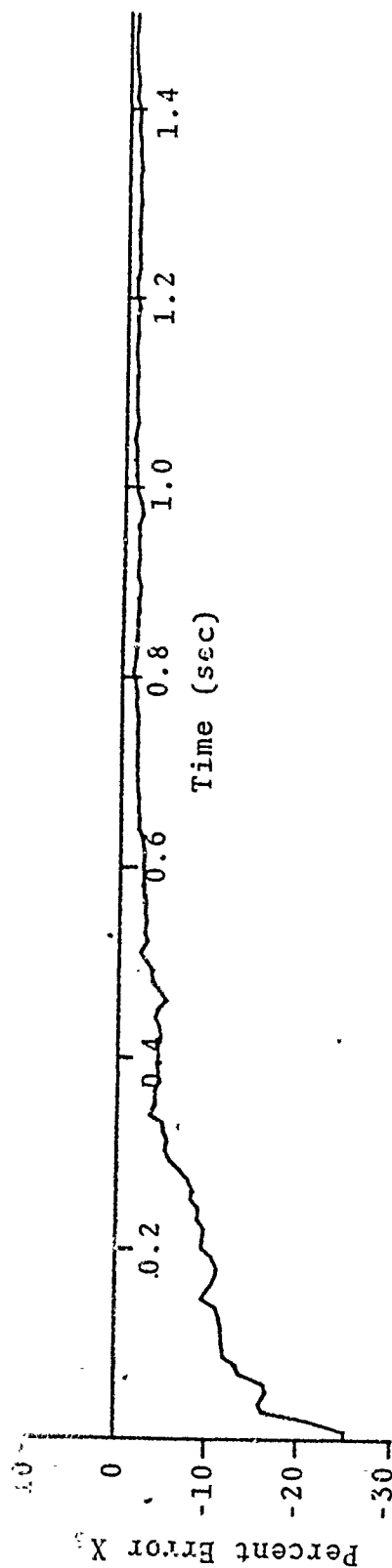
Figure 44. Variation of Percent Error in $\hat{X}_5$ with Time for Large Error in Initial Estimate of X_5 (Nonlinear System, Measurement Errors Only)
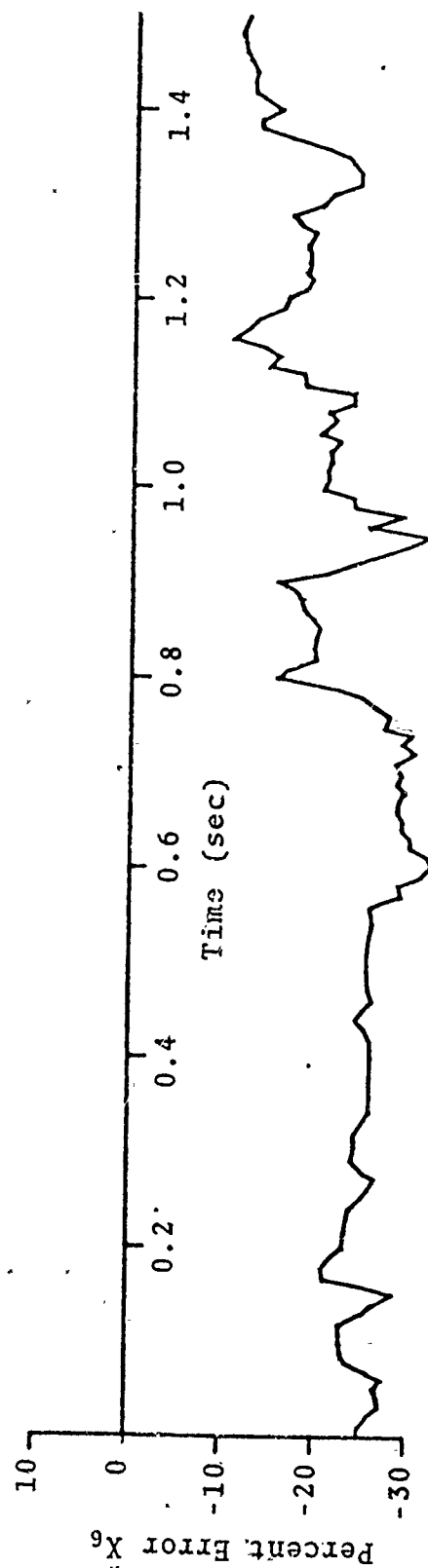
Figure 45. Variation of Percent Error in $\hat{X}_6$ with Time for Large Error in Initial Estimate of X_5 (Nonlinear System, Measurement Errors Only)

86
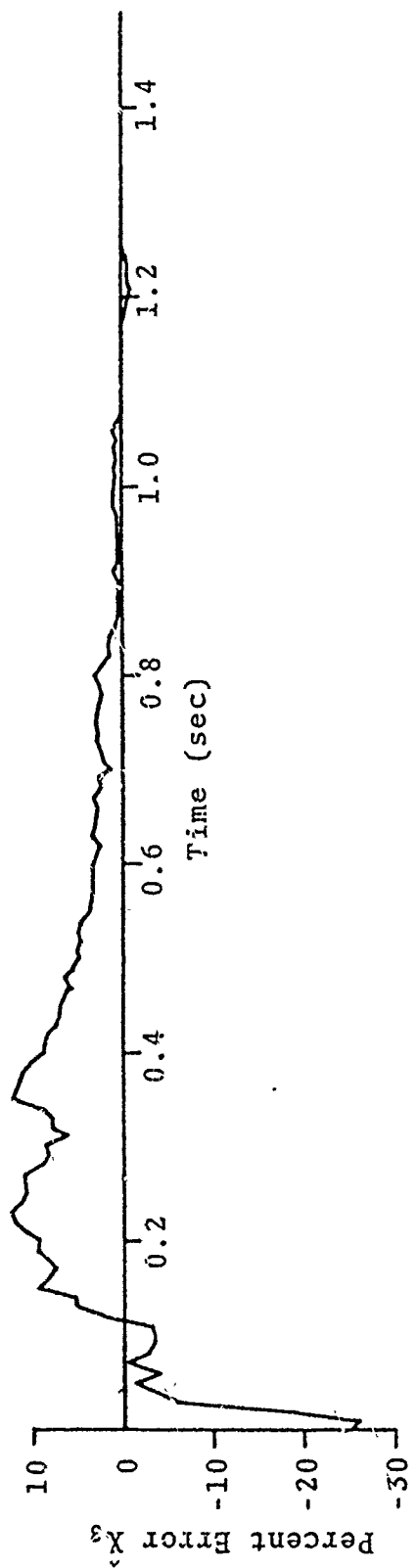
Figure 46.   Variation of Percent Error in $\hat{X}_3$ with Time (Nonlinear System, Measurement Errors and System Noise)
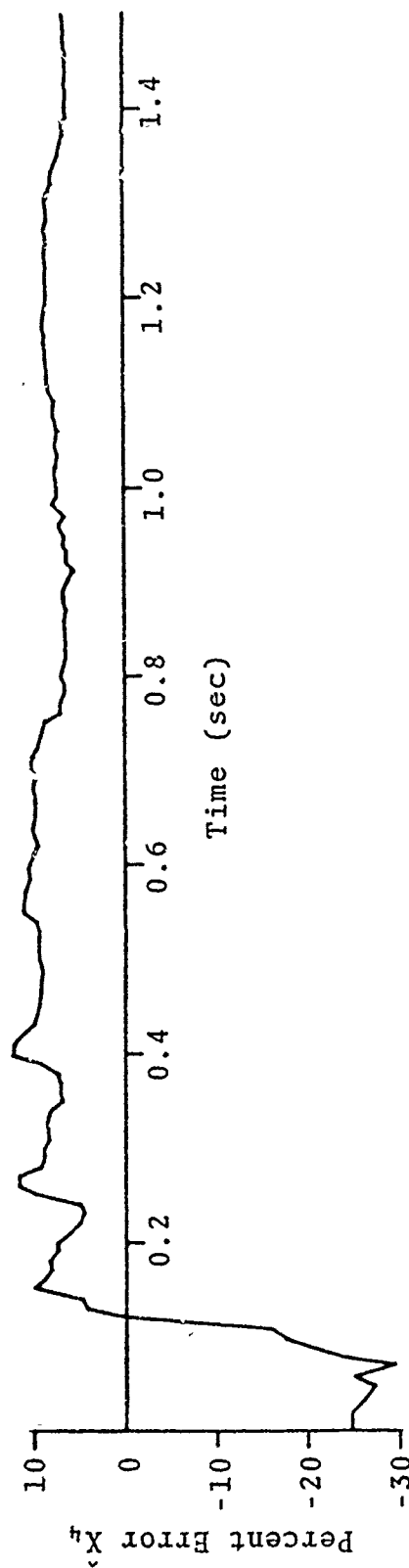
Figure 47.   Variation of Percent Error in $\hat{X}_4$ with Time (Nonlinear System, Measurement Errors and System Noise)

87
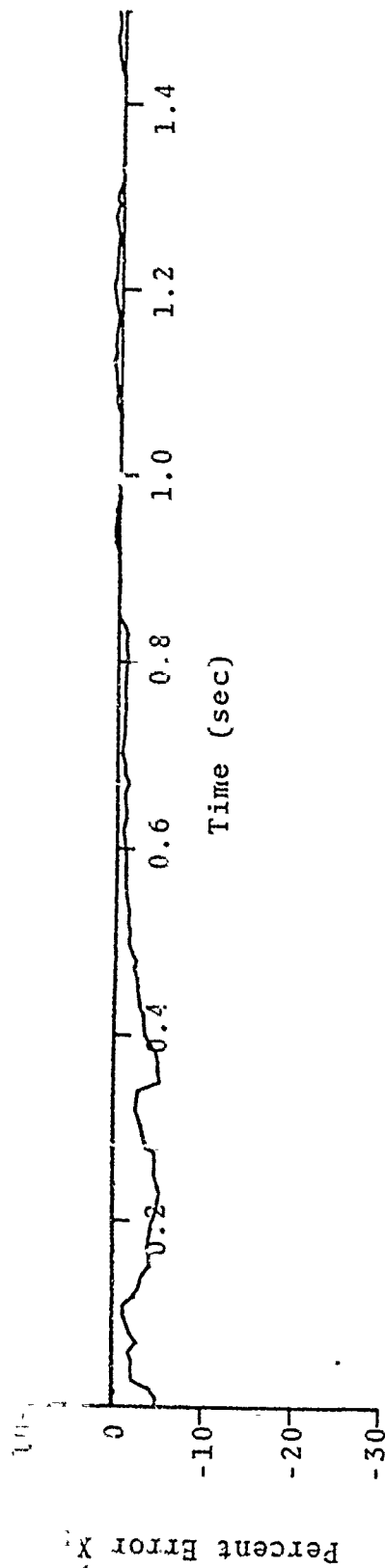
Figure 48. Variation of Percent Error in $\hat{X}_5$ with Time (Nonlinear System, Measurement Errors and System Noise)
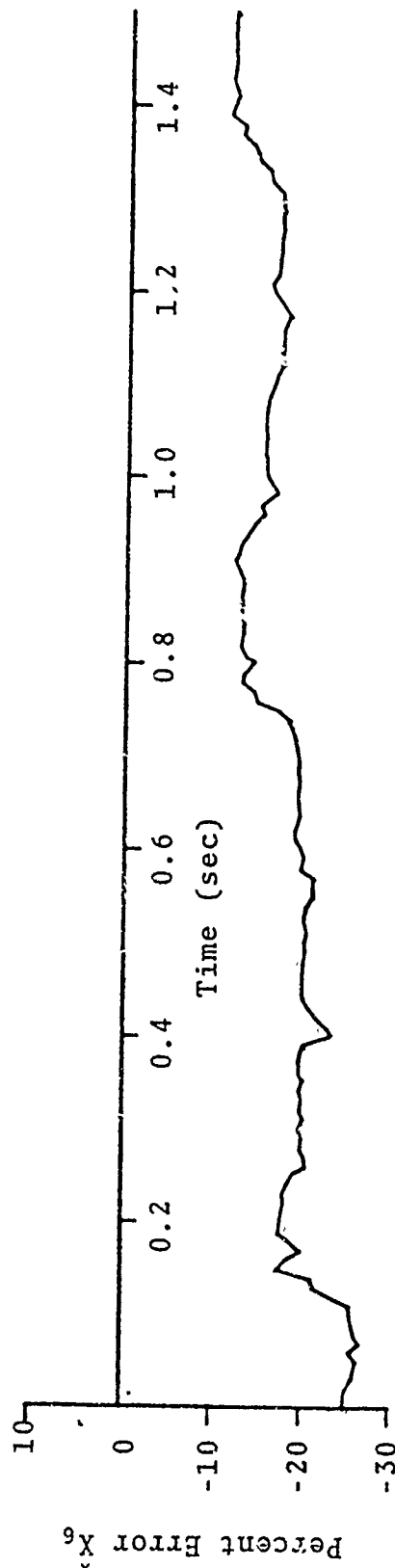
Figure 49. Variation of Percent Error in $\hat{X}_6$ with Time (Nonlinear System, Measurement Errors and System Noise)

88
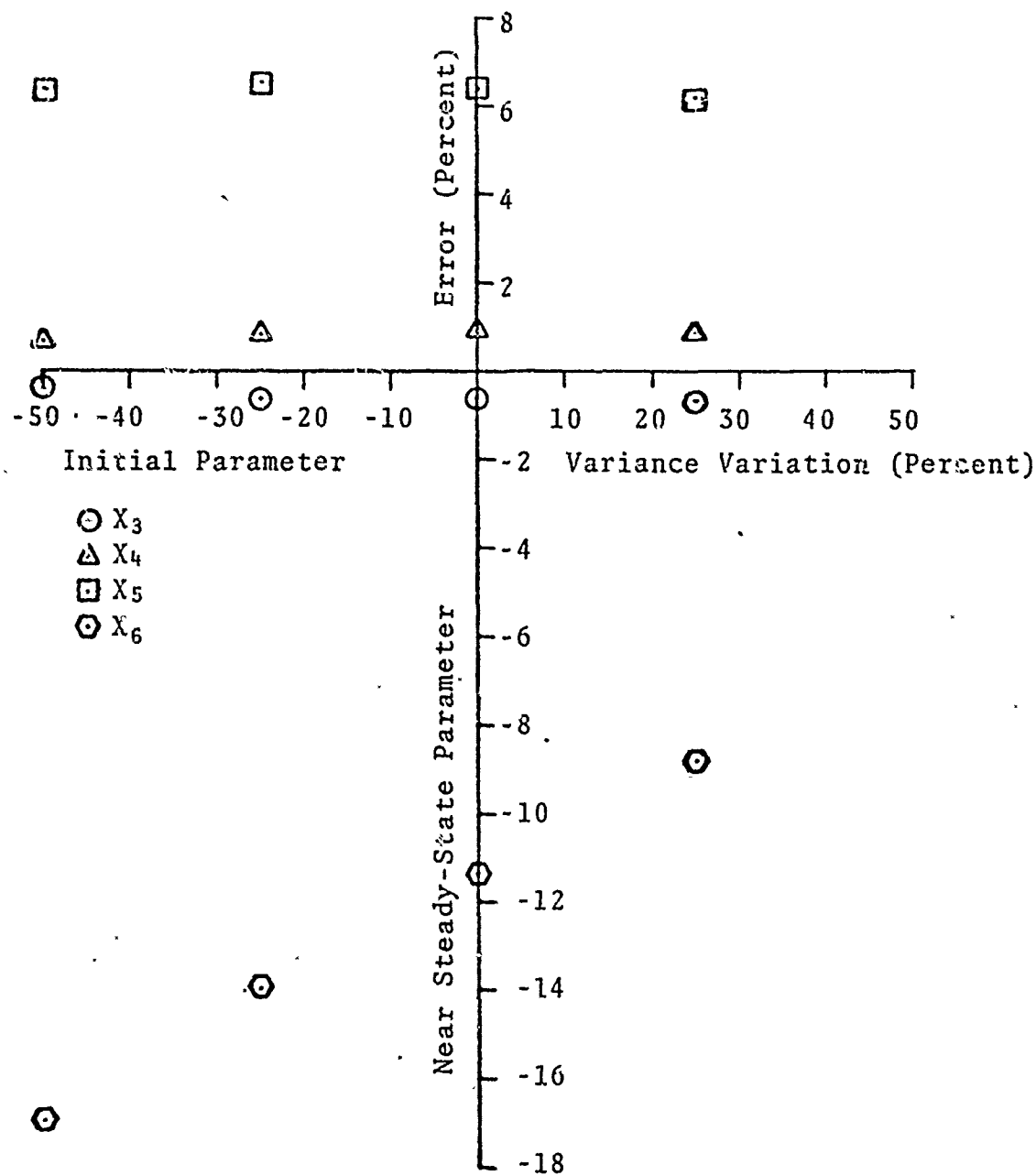
Figure 50. Variation of Near Steady-State Parameter Error
with Initial Parameter Variance (Nonlinear
System, Measurement Errors and System Noise)
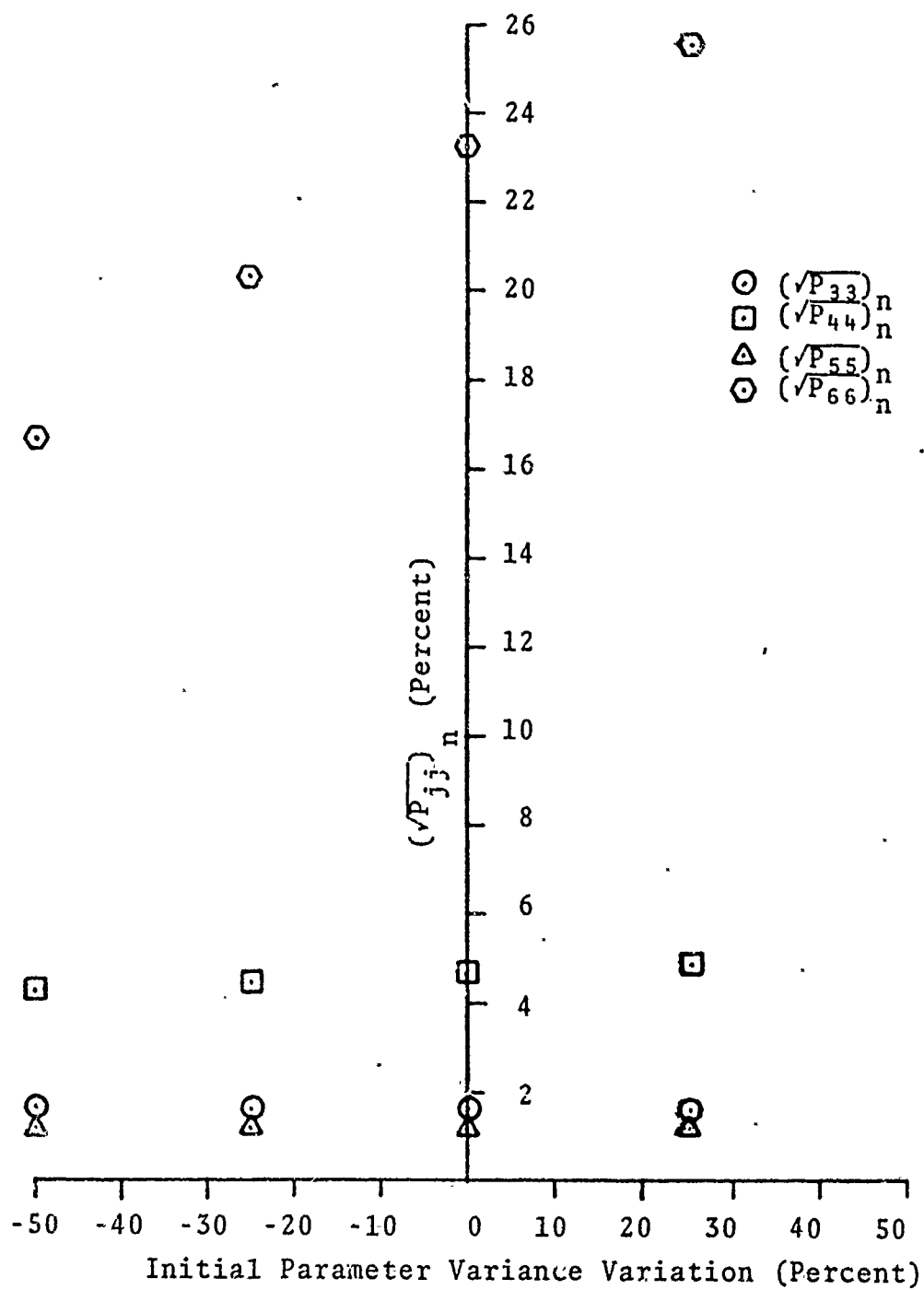
Figure 51.   Variation of Normalized Near Steady-State
              Parameter Uncertainty with Initial Parameter
              Variance (Nonlinear System, Measurement
              Errors and System Noise)
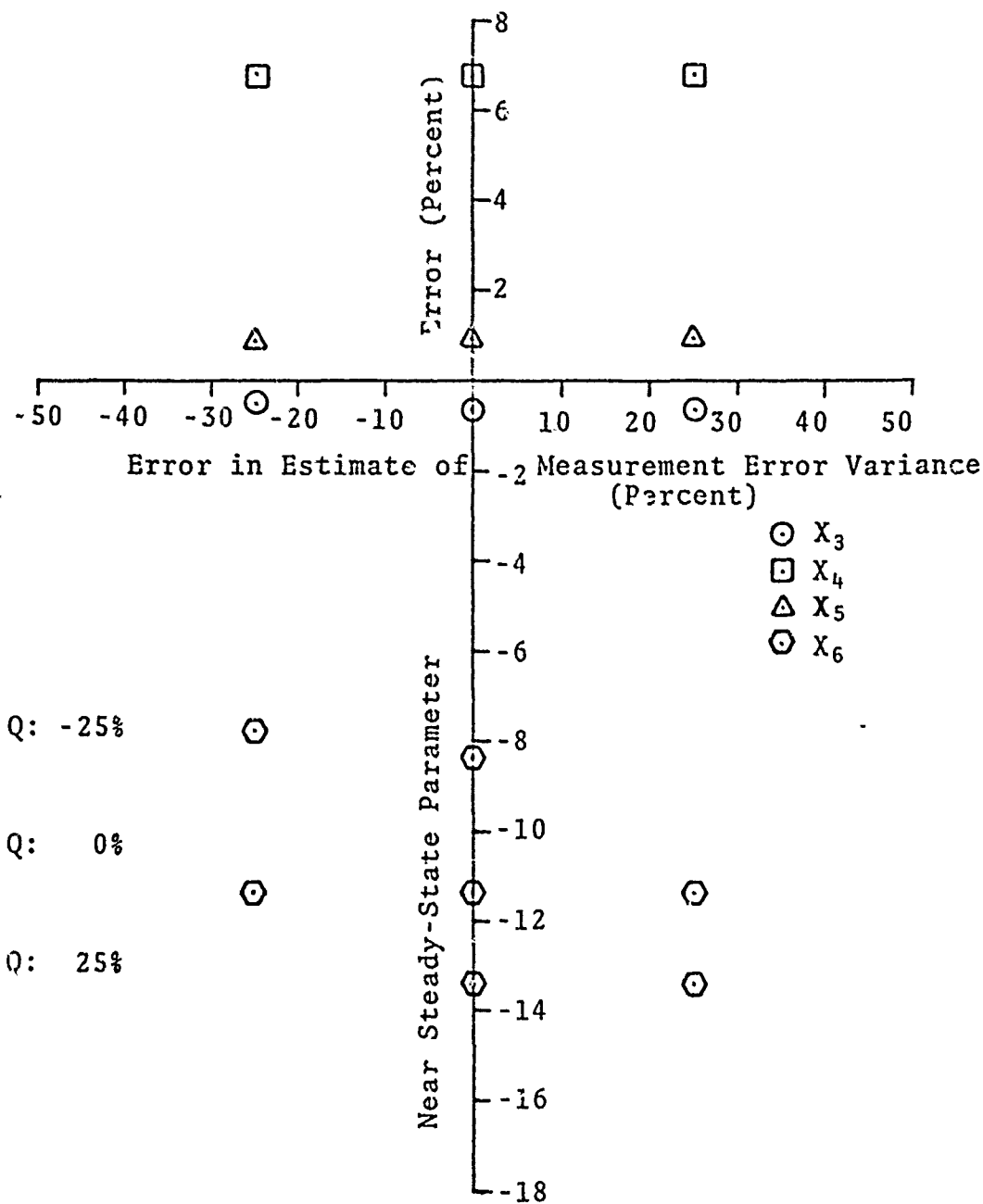
Figure 52. Variation of Near Steady-State Parameter Error with Error in the Estimate of the Measurement Error Variance (Nonlinear System, Measurement Errors and System Noise)
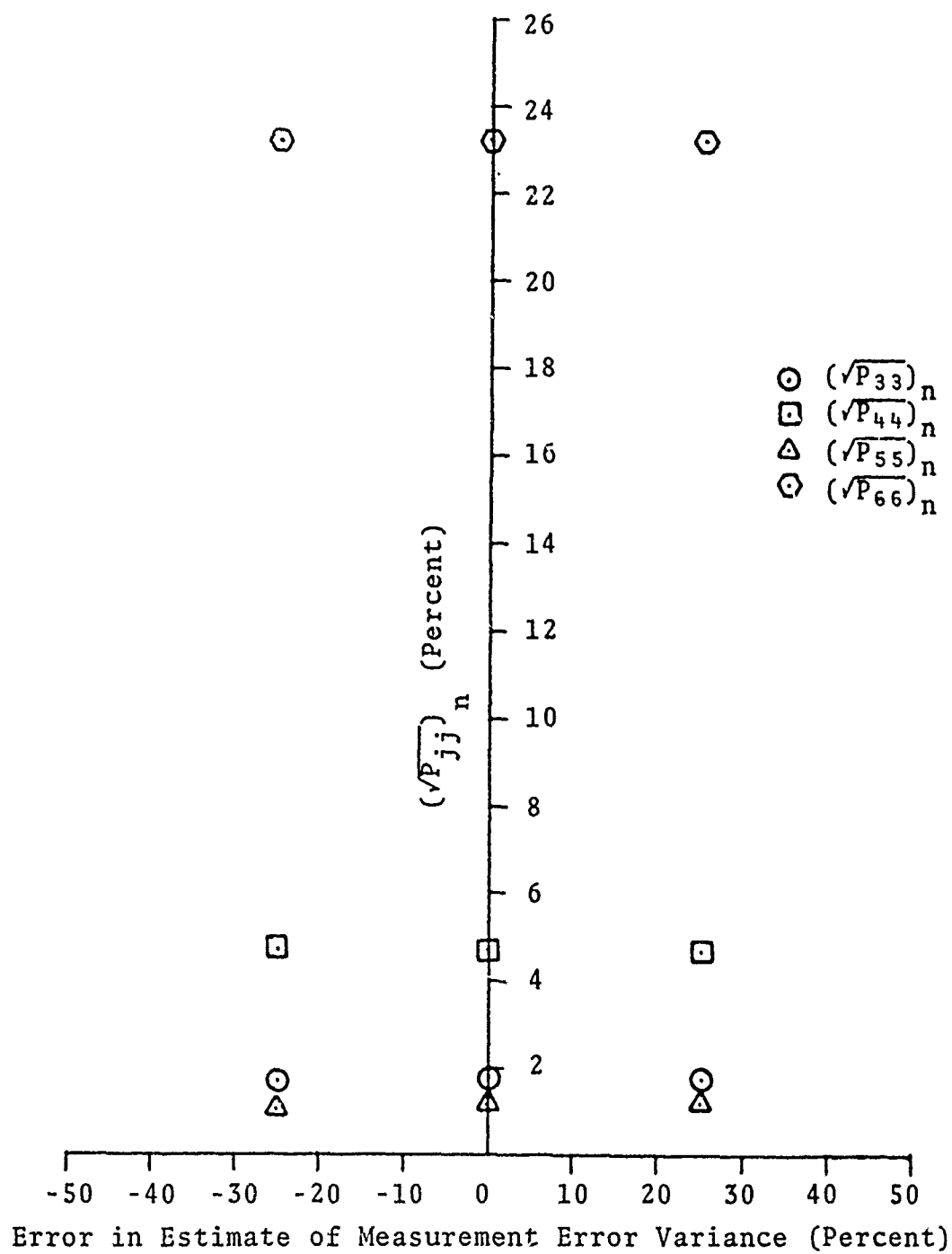
Figure 53. Variation of Normalized Near Steady-State
Parameter Uncertainty with Error in the
Estimate of Measurement Error Variance
(Nonlinear System, Measurement Errors and
System Noise)

SECTION VI

SUMMARY

1. General Comments

An analysis of methods for extracting aerodynamic coefficients from dynamic test data has been conducted. The primary concern of this analysis has been to determine the accuracy with which these coefficients can be determined from dynamic data containing system noise and measurement errors. Both linear and nonlinear systems have been considered with emphasis on the latter.

Two methods of extracting the coefficients of interest and estimating their uncertainties have been analyzed. The first of these is a deterministic technique due to Chapman and Kirk.[10] The second technique considered, based on the extended Kalman filter[20,31] with augmented state vector, is stochastic in nature and has been developed and applied in this report.

Both techniques considered will extract aerodynamic coefficients from noisy dynamic data. The degree to which they are successful has been presented in detail in the preceding chapters.

2. The Chapman-Kirk Technique

The presence of noise in the dynamic data being used by the Chapman-Kirk technique affects the accuracy with which this technique can be used to determine the coefficients of interest. When measurement errors only are present in the data, these effects have been found to be negligible on linear and nonlinear static pitching moment coefficient derivatives; the effects on the linear and nonlinear pitch damping coefficients are more pronounced, especially for the higher measurement errors. The error in the linear pitch damping coefficient is as high as 25 percent in one case. The greatest error in the extracted nonlinear pitch damping coefficient is 227 percent. There is an obvious inconsistency in the extracted values of this nonlinear coefficient since another data set, which also contained measurement errors with similar statistical properties, yielded a value for this term which was in error by only 5 percent.

The estimation of the uncertainties of the extracted coefficients when the data contained only measurement errors is quite adequate. In every case considered, the true value

93

of the coefficient of interest was found to be within three
estimated standard deviations of the extracted value. The
estimated standard deviations of a given coefficient are also
consistent for different data sets with the same measurement
error statistics.

Use of data containing measurement errors and system
noise has a more adverse effect on the accuracy of coeffici-
ents determined with the Chapman-Kirk technique. As is the
case when analyzing data containing only measurement errors,
the pitch damping coefficients are the ones most affected.
The errors in these coefficients range from 1 percent to
43 percent for the linear term and from 45 percent to 600
percent for the nonlinear term over the range of system noise
considered.

Perhaps of even greater consequence than the errors in
the damping parameters is the fact that their estimated
uncertainties are too small to indicate the true errors in
the coefficients when the data contain both system noise and
measurement errors. This is a direct consequence of using
the root-mean-square-error between the final solution to
the equation of motion and the data to calculate the parameter
uncertainties. This error term is an adequate representation
of noise in the data only if all errors are randomly super-
imposed on the pitch angle output after the equation of
motion has been integrated. The integrated effects of the
random accelerations representing system noise, however,
are such that the final errors appearing in the pitch angle
due to the system noise are not necessarily random or
independent from one discrete time to another.

3. Extended Kalman Filter

The feasibility of using an extended Kalman filter with
parameter augmented state vector for determining the values
of aerodynamic coefficients and their uncertainties from
dynamic test data has been demonstrated for a one-degree-
of-freedom system.

The specific filter used here generally reaches near
steady-state estimates of the parameters in less than one
second for the system model and error combinations considered.

For linear systems with measurement errors only in the data, the
extended filter yielded estimates of both the static pitching moment
coefficient derivative and the pitch damping coefficient to within 1
percent of their respective true values. Slightly less accurate deter-
minations of the same two parameters are obtained using the filter when
both measurement errors and system noise are present in the data.
Variations in the initial parameter variances of 25 percent seem to affect
essentially only the pitch damping term and then only when both measure-
ment errors and system noise are present in the data.

94

For nonlinear systems with measurement errors only in the data
the estimates obtained with the extended filter for the linear and
nonlinear static pitching moment coefficient derivatives as well as
the linear pitch damping coefficient are within approximately 1 per-
cent or less of their respective true values for a variety of initial
parameter variances and noise variance estimates. The
error in the estimate of the nonlinear damping coefficient
varies between 11 percent and 14 percent. None of the
coefficients are particularly sensitive to variations in
the initial parameter variances or noise variances con-
sidered. As is the case for the linear system, the
extended filter is less accurate in its parameter identi-
fication when both measurement errors and system noise
of the magnitudes considered here are present in the data.
The linear damping coefficient is most affected by
having the addition of system noise in the data. The
error in this coefficient increases from approximately
1 percent to approximately 6 percent for the cases con-
sidered. Errors in the noise variance estimates of
±25 percent have little or no appreciable effect on the
linear and nonlinear static pitching moment coefficient
derivatives or the linear pitch damping coefficient, and
only a slight effect on the nonlinear pitch damping
coefficient.

For the range of initial parameter errors, initial
parameter variances, and noise variances considered, the
extended Kalman filter produces an excellent estimate of
the parameter uncertainties after integrating the Ricatti
equations for approximately 1.5 to 2.0 seconds. For every
case considered where the initial parameter estimates are
within 25 percent of their true values, the near steady-
state parameter value is within two standard deviations
of its true value, and in the majority of cases within
one standard deviation, when the standard deviation is
taken to be the square root of the near steady-state
parameter variance.

4. Comparison of the Two Techniques

There is a natural tendency when analyzing two tech-
niques to compare the results achieved when solving similar
problems. Both techniques investigated have strengths and
weaknesses.

From the preceding discussion in this report, it is
obvious that the Chapman-Kirk technique requires less
information initially than the extended Kalman filter
since it is only necessary to make initial estimates of
the parameters in the former whereas the parameters, their
variances, and the noise variances must all be estimated
initially when using the extended filter.

There is no provision to weigh the initial parameter
estimates when using the Chapman-Kirk technique whereas

this is done through the initial variances when using
the extended filter. This feature is particularly
important if certain parameters are known with signifi-
cantly higher accuracy before the extraction process begins.
The Chapman-Kirk technique will sometimes actually down-
grade good initial estimates when analyzing noisy data
whereas the extended filter will tend to improve the
estimate or at least not downgrade it for the initial
parameter variances considered.

From an economic point of view, both techniques are
essentially equal concerning computational costs. The
extended filter must numerically integrate more equations
but only has to do this once; the Chapman-Kirk technique,
being an iterative scheme, must integrate fewer equations
more times.

The programming requirements of the extended filter
are less than those of the Chapman-Kirk technique since
the former requires no matrix inversions, simultaneous
equation solutions, or logic for including or excluding
certain parameters from the extraction process.

5.   Areas for Additional Work

This research is part of a larger and continuing
program whose goal is a critical study of methods for
extracting aerodynamic coefficients from dynamic data.
Since the research is of a continuing nature, it seems
appropriate to mention some areas where additional work
should be contemplated.

The results presented here are entirely for a one-
degree-of-freedom system. The effect of system noise and
measurement errors on coefficients extracted from coupled,
multiple-degree-of-freedom systems may prove to be
extremely interesting and is the next logical area to be
investigated.

The response of the extended filter to highly erroneous
initial parameter estimates seems quite good for the limited
number of cases investigated. It seems possible to take
advantage of this feature by constructing an extraction
algorithm which employs both an extended filter and a
Chapman-Kirk or similar technique in tandem with the
extended filter providing the required accurate initial
estimates for the differential corrections, iterative
technique.

Additional work in relating the effects of inadequate
or erroneous modeling to the noise problem should also
prove to be of interest.

Finally, when considering new techniques such as the use of the extended Kalman filter, it must be concluded that additional numerical investigations will add to the confidence or more clearly define the limitations of this method of coefficient extraction.

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APPENDIX I

CHAPMAN-KIRK COEFFICIENT EXTRACTION NOMENCLATURE
LIST AND PROGRAM LISTING

Nomenclature List

PROGRAM VARIABLE	MATH SYMBOL	DEFINITION
H	Δt	Numerical integration step size (sec)
ITO		Frequency of numerical integration output
TMAX	t_{max}	Cutoff time for numerical integration (sec)
TZERO	t_0	Initial time for numerical integration (sec)
N	N	Number of first order differential equations to be integrated
XZ(1)	α_0	Initial condition of α (rad)
XZ(2)	$\dot{\alpha}_0$	Initial condition of $\dot{\alpha}$ (rad/sec)
XZ(3)	$\left.\dfrac{\partial \alpha}{\partial c_1}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(4)	$\left.\dfrac{d}{dt}\dfrac{\partial \alpha}{\partial c_1}\right|_{t=0}$	Initial condition for parametric differntial equation
XZ(5)	$\left.\dfrac{\partial \alpha}{\partial c_2}\right|_{t=0}$	Initial condition for parametric differential equation

PROGRAM VARIABLE	MATH SYMBOL	DEFINITION
XZ(6)	$\dfrac{d}{dt}\left.\dfrac{\partial \alpha}{\partial c_2}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(7)	$\left.\dfrac{\partial \alpha}{\partial c_3}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(8)	$\dfrac{d}{dt}\left.\dfrac{\partial \alpha}{\partial c_3}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(9)	$\left.\dfrac{\partial \alpha}{\partial c_4}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(10)	$\dfrac{d}{dt}\left.\dfrac{\partial \alpha}{\partial c_4}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(11)	$\left.\dfrac{\partial \alpha}{\partial c_5}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(12)	$\dfrac{d}{dt}\left.\dfrac{\partial \alpha}{\partial c_5}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(13)	$\left.\dfrac{\partial \alpha}{\partial c_6}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(14)	$\dfrac{d}{dt}\left.\dfrac{\partial \alpha}{\partial c_6}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(15)	$\left.\dfrac{\partial \alpha}{\partial c_7}\right|$	Initial condition for parametric differential equation
XZ(16)	$\dfrac{d}{dt}\left.\dfrac{\partial \alpha}{\partial c_7}\right|_{t=0}$	Initial condition for parametric differential equation

PROGRAM VARIABLE	MATH SYMBOL	DEFINITION
XZ(17)	$\left.\dfrac{\partial \alpha}{\partial c_8}\right|_{t=0}$	Initial condition for parametric differential equation
XZ(18)	$\left.\dfrac{d}{dt}\dfrac{\partial \alpha}{\partial c_6}\right|_{t=0}$	Initial condition for parametric differential equation
Q	q	Dynamic pressure (lb/ft^2)
V	v	Freestream velocity (ft/sec)
AMI	I	Moment of inertia about an axis through the vehicle C.G. and normal to the pitch plane $(slug \cdot ft^2)$
A	A	Vehicle reference area (ft^2)
D	d	Vehicle reference length (ft)
CMAO	$C_{m\alpha o}$	Static pitching moment coefficient derivative (rad^{-1})
CMA2	$C_{m\alpha 2}$	Static pitching moment coefficient derivative (rad^{-3})
CMA4	$C_{m\alpha 4}$	Static pitching moment coefficient derivative (rad^{-5})
CMQO	C_{mqo}	Pitch damping coefficient
CMQ2	C_{mq2}	Pitch damping coefficient (rad^{-2})
CMDA	$C_{m\delta\alpha}$	Static pitching moment coefficient at $\alpha = 0$

PROGRAM VARIABLE	MATH SYMBOL	DEFINITION
M	M	Number of experimental data points
JJJMAX		Maximum number of iterations allowed before program termination
EP	ε	Convergence criteria for change in RMS ERROR (rad)
AE(I)	$\alpha_{exp}(t_i)$	Experimental values of α (rad)
C1	C_1	α_o
C2	C_2	$\dot{\alpha}_o$
C3	C_3	$-\dfrac{C_{m\alpha o}q\ Ad}{I}$
C4	C_4	$-\dfrac{C_{mqo}q\ Ad^2}{2VI}$
C5	C_5	$-\dfrac{C_{m\alpha_2}q\ Ad}{I}$
C6	C_6	$-\dfrac{C_{mq_2}q\ Ad^2}{2VI}$
C7	C_7	$-\dfrac{C_{m\alpha_4}q\ Ad}{I}$
C8	C_8	$-\dfrac{C_{m\delta\alpha}q\ Ad}{I}$

Categories of Input Data

Numerical integration constants.--These constants include the numerical integration step size, frequency of output, time at which the numerical integration is to stop, initial time, and the number of first order equations to be integrated.

Initial conditions.--These data are the first estimates of the initial conditions for the equation of motion as well as the initial conditions for the parametric differential equations.

Aerodynamic and physical constants.--These constants are the dynamic pressure and freestream velocity that existed when the experimental pitch angle data were recorded, the moment of inertia about an axis normal to the pitch plane and through the center of gravity of the vehicle of interest, a vehicle reference area, and a vehicle reference length.

Aerodynamic coefficient estimates.--These data are the first estimates of the aerodynamic coefficients to be extracted.

Convergency criteria.--These constants are the number of experimental data points to be read, the maximum number of iterations to be allowed, and the tolerance against which the root-mean-square error difference is tested for convergence.

Experimental data.--These data are the experimental pitch angle values from which the aerodynamic coefficients are to be extracted. The program assumes the time between each data point is constant, and this time must be equal to the product of the numerical integration step size and frequency of numerical integration output that were given on the first data card.

```
      DIMENSION XZ(18),AE(500),X(120),PC(8,500),AC(500),
     1AJK(4,4),B(8),DELC(8),L1(4),L2(4),DIF(500),STDEV(8),
     2ACDOT(500)
      COMMON N,T,X
      COMMON C1,C2,C3,C4,C5,C6,C7,C8,KKK,M,PC,AC,ACDOT
      EXTERNAL XDOT,OUT
      JJJ=0
      READ(5,1) H,ITO,TMAX,TZERO,N
    1 FORMAT (F5.3,I3,2F5.3,I3)
      READ(5,2) (XZ(I),I=1,N)
    2 FORMAT(2F10.4)
      READ (5.3) Q,V,AMI,A,D,
    3 FORMAT (5F8.4)
      READ(5,4) CMAO,CMA2,CMA4,CMQO,CMQ2,CMDA
    4 FORMAT(6F10.3)
      IF(N.EQ.2) M=TMAX/H
      IF (N.EQ.2) GO TO 55
      READ(5,41) M,JJJMAX,EP
   41 FORMAT(2I3,E10.3)
      READ(5,5) (AE(I),I=1,M)
    5 FORMAT(10F8.5)
   55 CONTINUE
      CON1=(-Q*A*D**2/(2.0*V*AMI))
      CON2=(-Q*A*D/AMI)
      C1=XZ(1)
      C2=XZ(2)
      C3=CON2*CMAO
      C4=CON1*CMQO
      C5=CON2*CMA2
      C6=CON1*CMQ2
      C7=CON2*CMA4
      C8=CON2*CMDA
      WRITE(6,59)
   59 FORMAT('0',5X,'NUMERICAL INTEGRATION CONSTANTS')
```

104

```
      WRITE(6,60) H,ITO,TMAX,TZERO,N
   60 FORMAT('0',5X,'H(SEC)=',F5.3,5X,'ITO=',I3,5X,'TMAX(SEC)=',
     1F5.3,5X,'TZERO(SEC)=',F5.3,5X,'N=',I3)
      WRITE(6,61)
   61 FORMAT('0',5X,'AERODYNAMIC INPUT DATA')
      WRITE(6,6) Q,V,AMI,A,D
    6 FORMAT('0',5X,'Q(LB/FT**2)=',F8.4,5X,'V(FT/SEC)=',F8.4,5X,
     1'AMI(SLG*FT**2)=',F8.4,5X,'A(FT**2)=',F8.4,5X,'D(FT)=',
     22F8.4)
      WRITE(6,62)
   62 FORMAT('0',5X,'AERODYNAMIC COEFFICIENT ESTIMATES')
      WRITE(6,7)CMAO,CMA2,CMA4,CMQO,CMQ2,CMDA,M
    7 FORMAT('0',5X,'CMAO=',F9.3,5X,'CMA2=',F9.3,5X,'CMA4=',F9.3,
     15X,'CMQO=',F9.3,5X,'CMQ2=',F9.3,5X,'CMDA=',F9.3,5X,'M=',I3)
      IF(N.EQ.2) GO TO 88
      WRITE(6,63)
   63 FORMAT('0',5X,'EXPERIMENTAL ANGLE OF ATTACK DATA')
      WRITE(6,8)  (I,AE(I),I=1,M)
    8 FORMAT('0',5X,'AE(',I3,')=',F8.5,5X,'AE(',I3,')=',F8.5,5X,
     1'AE(',I3,')=',F8.5,5X,'AE(',I3,')=',F8.5,5X,'AF(',I3,')=',
     22F8.5)
   88 CONTINUE
      KKK=0
      CALL ADDUM(H,ITO,TZERO,TMAX,XZ,XDOT,OUT)
      IF(N.EQ.2) GO TO 148
      IF(JJJ.GT.O) GO TO 880
      WRITE(6,879)
  879 FORMAT('1',5X,'CURRENT PARAMETER VALUES')
      GO TO 8881
  880 WRITE(6,881)
  881 FORMAT('0',5X,'CURRENT PARAMETER VALUES')
 8881 WRITE(6,882)JJJ,C1,C2,C3,C4,C5,C6,C7,C8
  882 FORMAT('0',5X,'JJJ=',I2,5X,'C1=',E12.5,5X,'C2=',E12.5,5X,
     1'C3=',E12.5,5X,'C4=',E12.5,//,17X,'C5=',E12.5,5X,'C6=',
```

105

```
2E12.5 5X, '9' '=', E12.5, 5X, 'C8=', E12.5)
      NN= (N-2)/2
      DO 890    J=1,8
      DELC(J)=0.
      STDEV(J)=0.
890   CONTINUE
      DO 89 J=1,NN
      B(J)=0.
      DO 89 K=1,NN
      AJK(J,K)=0.
89    CONTINUE
      DO 9 L=1,M
      DO 9 J=1,NN
      DIF(L)=AE(L)-AC(L)
      B(J)=B(J)+PC(J,L)*DIF(L)
      DO 9 K=1,J
      AJK(J,K)=AJK(J,K)+PC(J,L)*PC(K,L)
      IF(J.EQ.K) GO TO 9
      AJK(K,J)=AJK(J,K)
9     CONTINUE
      WRITE(6,900)  ((AJK(J,K),K=1,NN),J=1,NN)
900   FORMAT(6E15.5)
      CALL MINV(AJK,NN,R,L1,L2)
      WRITE(6,902)  ((AJK(J,K),K=1,NN),J=1,NN)
902   FORMAT(6E15.5)
      WRITE(6,901)(B(J),J=1,NN)
901   FORMAT(6E15.5)
      R=0.
      DO 883 L=1,M
      R=R+DIF(L)*DIF(L)
883   CONTINUE
      IF(JJJ.GT.0) GO TO 91
      RMSE=0.
91    RMSEP=RMSE
```

106

```
      RMSR=SQRT(R/M)
      RMSE=SQRT(R/(M-NN))
      DIFF=ABS(RMSE-RMSEP)
      DO 910 J=1,NN
      STDEV(J)=RMSE*(SQRT(AJK(J,J)))
  910 CONTINUE
      WRITE(6,92)    RMSR
   92 FORMAT('0',5X,'RMS RESIDUAL=',E16.8)
      WRITE(6,93)  RMSE
   93 FORMAT('0',5X,'RMS ERROR=',E16.8)
      WRITE(6,94)
   94 FORMAT('0',5X,
     1'ESTIMATED STANDARD DEVIATIONS ·OF THE PARAMETERS')
      WRITE(6,95)  STDEV
   95 FORMAT('0',16X,'SDC1=',E12.5,5X,'SDC2=',E12.5,5X,'SDC3=',
     1E12.5,  5X,'SDC4=',E12.5,//,17X,'SDC5-',E12.5,5X,'SDC6=',
     2E12.5,5X,'SDC7=',E12.5,5X,'SDC8=',E12.5)
      IF(DIFF.LT.EP) GO TO 13
      IF(JJJ.GE.JJJMAX) GO TO 121
      JJJ=JJJ*1
      DO 10 I=1,NN
      DO 10 J=1,NN
   10 DELC(I)=DELC(I)+AJK(I,J)*B(J)
      WRITE(6,101)
  101 FORMAT('0',5X,'CURRENT PARAMETER CORRECTIONS')
      WRITE(6,102)  JJJ,DELC
  102 FORMAT('0',5X,'JJJ=',I2,5X,'DELC1=',E12.5,5X,'DELC2=',
     1E12.5,5X,'DELC3=',E12.5,5X,'DELC4=',E12.5,//,17X,'DELC5=',
     2E12.5,5X,'DELC6=',E12.5,5X,'DELC7=',E12.5,5X,'DELC8=',
     3E12.5)
      GO TO 1222
  121 WRITE(6,122)
  122 FORMAT('0              CONVERGENCE FAILED - MAXIMUM NUMBER OF ',
     1 'ITERATIONS EXCEEDED')
```

107

```
      GO TO 5
1222  C1=C1+DELC(1)
      C2=C2+DELC(2)
      C3=C3+DELC(3)
      C4=C4+DELC(4)
      C5=C5+DELC(5)
      C6=C6+DELC(6)
      C7=C7+DELC(7)
      C8=C8+DELC(8)
      XZ(1)=C1
      XZ(2)=C2
      GO TO 88
13    CONTINUE
      CMAO=C3/CON2
      CMQO=C4/CON1
      CMA2=C5/CON2
      CMQ2=C6/CON1
      CMA4=C7/CON2
      CMDA=C8/CON2
      SDCMAO=STDEV(3)/(-CON2)
      SDCMQO=STDEV(4)/(-CON1)
      SDCMA2=STDEV(5)/(-CON2)
      SDCMQ2=STDEV(6)/(-CON1)
      SDCMA4=STDEV(7)/(-CON2)
      SDCMDA=STDEV(8)/(-CON2)
      WRITE(6,139)
139   FORMAT ('1           EXTRACTED INITIAL CONDITIONS AND ',
     1 'AERODYNAMIC COEFFICIENTS')
      WRITE(6,14) C1,C2
14    FORMAT('0',5X,'ALPHAO=',F6.4,5X,'ALDOTO=',F6.4)
      WRITE(6,140) STDEV(1),STDEV(2)
140   FORMAT('0',5X,'SDA=',F9.4,5X,'SDADOT=',F6.4)
      WRITE(6,15) CMAO,CMA2,CMA4,CMQO,CMQ2,CMDA
15    FORMAT('0',5X,'CMAO=',F9.3,5X,'CMA2=',F9.3,5X,'CMA4=',
```

```
      1F9.3,5X,'CMQO=',F9.3,5X,'CMQ2=',F9.3,5X,'CMDA=',F9.3)
      WRITE(6,147) SDCMAO,SDCMA2,SDCMA4,SDCMQO,SDCMQ2,SDCMDA
  147 FORMAT('0',5X,'SDCMAO=',F7.3,5X,'SDCMA2=',F7.3,5X,
     1'SDCMA4=',F7.3,5X,'SDCMQO=',F7.3,5X,'SDCMQ2=',F7.3,5X,
     2'SDCMDA=',F7.3)
  148 CONTINUE
      IF(N.GT.2) GO TO 1491
      WRITE(7,1490) (AC(I(,I=1,M)
 1490 FORMAT(10F8.5)
 1491 CONTINUE
      WRITE(6,149)
  149 FORMAT('0',5X,'CALCULATED ANGLE OF ATTACK OUTPUT')
      WRITE(6,150) (I,AC(I),I=I,M)
  150 FORMAT('0',5X'AC(',I3,')=',F8.5,5X,'AC(',I3,')=',F8.5,5X
     1'AC(',I3,')=',F8.5,5X,AC(',I3,')=',F8.5,5X,'AC(',I3,')=',
     22F8.5)
   16 CONTINUE
      STOP
      END
```

109

```
      SUBROUTINE ADDUM(AITCH,ITO,TZERO,TMAX,X,F,OUT)        ADDU 100
      DIMENSION XZ(20),C(20,6),X(20,6)                      ADDU 110
      COMMON N,T,X,                                         ADDU 120
      H = ABS(AITCH)                                        ADDU 130
      IT = ITO-1                                            ADDU 140
      D=TZERO-TMAX                                          ADDU 150
      IF (D)2,1,1                                           ADDU 160
    1 H=-H                                                  ADDU 170
   .2 HH=0.5*H                                              ADDU 180
      D=H/24.0                                              ADDU 190
      T=TZERO                                               ADDU 200
      ISET=0                                                ADDU 210
      DO       3  I=1,N                                     ADDU 220
      X(I,1)=XZ(I)                                          ADDU 230
    3 X(I,6)=XZ(I)                                          ADDU 240
      CALL F(X(1,5),1)                                      ADDU 250
      CALL F(X(1,2),0)                                      ADDU 260
      IF(ITO)5,5,4                                          ADDU 270
    4    CALL OUT(1)                                        ADDU 280
    5 DO        6 K=1,N                                     ADDU 290
         C(K,1) = X(K,5)*HH                                 ADDU 300
    6    X(K,1) = X(K,6) + C(K.1)                           ADDU 310
      T=T+HH                                                ADDU 320
      CALL F(C(1,2),3)                                      ADDU 330
      DO        7 K=1,N                                     ADDU 340
         C(K,2) = HH*C(K,2)                                 ADDU 350
    7    X(K,1) = X(K,6) + C(K,2)                           ADDU 360
      CALL F(C(1,3),3)                                      ADDU 370
      DO        8 K=1,N                                     ADDU 380
         C(K,3) = H*C(K,3)                                  ADDU 390
    8    X(K,1) = X(K,6) + C(K,3)                           ADDU 400
      T=T+HH                                                ADDU 410
      CALL F(C(1,4),1)                                      ADDU 420
      DO        9 K=1,N
```

110

```
      C(K,4) = C(K,4)*H                                            ALDU 430
      X(K,1)=X(K,6)+(C(K,1)+2.0*C(K,2)+C(K,3)+0.5*C(K,4))          ADDU 450
     1                    *0.33333333                              ADDU 460
    9 X(K,6) = X(K,1)                                              ADDU 470
      ISET = ISET+1                                                ADDU 480
      GO TO (10,12,17),ISET                                        ADDU 490
   10 CALL F(X(1,5),0)                                             ADDU 500
      DO   11 K = 1,N                                              ADDU 510
   11 X(K,3)=X(K,5)                                                ADDU 520
      GO TO   18                                                   ADDU 530
   12 CALL F(X(1,5),0)                                             ADDU 540
      DO   13 K=1,N                                                ADDU 550
   13 X(K,4)=X(K,5)                                                ADDU 560
      GO TO 18                                                     ADDU 570
   14 T=T+H
      DO   15 K=1,N                                                AD U 5 0
      X(K,1)=X(K,6)+D*(55.0*X(K,5)-59.0*X(K,4)+37.0*X(K,3)         ADDU 600
     1              -9.0*X(K,2))                                   ADDU 610
      X(K,2)=X(K,3)                                                ADDU 620
      X(K,3)=X(K,4)                                                ADDU 630
      X(K,4)=X(K,5)
   15 CALL F(X(1,5),1)
      DO   16 K = 1,N                                              ++ 4
      X(K,6)=X(K,6)+D*(9.0*X(K,5)+19.0*X(K,4)-5.0*X(K,3)           ADDU 670
     1              +X(K,2))                                       4
   16 X(K,1)=X(K,6)                                                ADDU 690
      ISET = 3                                                     ADDU 700
   17 CALL F(X(1,5),0)                                             ADDU 710
   18 IF (IT)20,19,20                                              ADDU 720
   19 IT = ITO                                                     ADDU 730
      CALL OUT(9)
   20 IT = IT - 1
      IF(ABS(TMAX-T)-ABS(HH))21,21,22
   21 RETURN
```

ADDU 750

22   GO TO (5,5 ...), SGT
  END

```
      SUBROUTINE XDOT(A,K)
      DIMENSION A(18),X(120),PC(8,500),AC(500),ACDOT(500)
      COMMON N,T,X
      COMMON C1,C2,C3,C4,C5,C6,C7,C8,KKK,M,PC,AC,ACDOT
      EQUIVALENCE (X(1),ALPHA),(X(2),ALDOT),(X(3),P1),(X(4),
     1P1DOT),(X(5),P2),(X(6),P2DOT),(X(7),P3),(X(8),P3DOT),(X(9),
     2P4),(X(10),P4DOT),(X(11),P5),(X(12),P5DOT),(X(13),P6),
     3(X(14),P6DOT),(X(15),P7),(X(16),P7DOT),(X(17),P8),(X(18),
     4P8DOT)
      A(1)=ALDOT
      A1=-(C4 + C6*ALPHA**2)*ALDOT
      A2=-(C3+C5*ALPHA**2+C7*ALPHA**4)*ALPHA
      A(2)=A1+A2-C8
      IF(N.EQ.2) GO TO 1
      A3=C4+C6*ALPHA**2
      A4=2*C6*ALPHA*ALDOT+C3+3*C5*ALPHA**2+5*C7*ALPHA**4
      A(3)=P1DOT
      A(4)=-A3*P1DOT-A4*P1
      A(5)=P2DOT
      A(6)=-A3*P2DOT-A4*P2
      A(7)=P3DOT
      A(8)=-A3*P3DOT-A4*P3-ALPHA
      IF(N.EQ.8) GO TO 1
      A(9)=P4DOT
      A(10)=-A3*P4DOT-A4*P4-ALDOT
      IF(N.EQ.10) GO TO 1
      A(11)=P5DOT
      A(12)=-A3*P5DOT-A4*P5-ALPHA**3
      IF(N.EQ.12) GO TO 1
      A(13)=P6DOT
      A(14)=-A3*P6DOT-A4*P6-(ALPHA**2)*ALDOT
      IF(N.EQ.14) GO TO 1
      A(15)=P7DOT
      A(16)=-A3*P7DOT-A4*P7-ALPHA**5
```

```
      IF(N.EQ.16) GO TO 1
      A(17)=P8DOT
      A(18)=-A3*P8DOT-A4*P8-1.0
    1 CONTINUE
      RETURN
      END
```

```
SUBROUTINE OUT(K)
DIMENSION X(120),PC(8,500),AC(500),ACDOT(500)
COMMON N,T,X
COMMON C1,C2,C3,C4,C5,C6,C7,C8,KKK,M,PC,AC,ACDOT
KKK=KKK+1
IF(N.EQ.2) GO TO 1
PC(1,KKK)=X(3)
PC(2,KKK)=X(5)
PC(3,KKK)=X(7)
IF(N.EQ.8) GO TO 1
PC(4,KKK)=X(9)
IF(N.EQ.10) GO TO 1
PC(5,KKK)=X(11)
IF(N.EQ.12) GO TO 1
PC(6,KKK)=X(13)
IF(N.EQ.14) GO TO 1
PC(7,KKK)=X(15)
IF(N.EQ.16) GO TO 1
PC(8,KKK)=X(17)
1 AC(KKK)=X(1)
ACDOT(KKK)=X(2)
RETURN
END
```

```
C     ..................................................................      MINV  10
C                                                                             MINV  20
C        SUBROUTINE MINV                                                      MINV  30
C                                                                             MINV  40
C        PURPOSE                                                              MINV  50
C           INVERT A MATRIX                                                   MINV  60
C                                                                             MINV  70
C        USAGE                                                                MINV  80
C           CALL MINV(A,N,D,L,)                                               MINV  90
C                                                                             MINV 100
C        DESCRIPTION OF PARAMETERS                                            MINV 110
C           A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY        MINV 120
C               RESULTANT INVERSE.                                            MINV 130
C           N - ORDER OF MATRIX A                                             MINV 140
C           D - RESULTANT DETERMINANT                                         MINV 150
C           L - WORK VECTOR OF LENGTH N                                       MINV 160
C           M - WORK VECTOR OF LENGTH N                                       MINV 170
C                                                                             MINV 180
C        REMARKS                                                              MINV 190
C           MATRIX A MUST BE A GENERAL MATRIX                                 MINV 200
C                                                                             MINV 210
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                        MINV 220
C           NONE                                                              MINV 230
C                                                                             MINV 240
C        METHOD                                                               MINV 250
C           THE STANDARD GAUSS-JORDAN METHOD IS USED.  THE DETERMINANT        MINV 260
C           IS ALSO CALCULATED.  A DETERMINANT OF ZERO INDICATES THAT         MINV 270
C           THE MATRIX IS SINGULAR.                                           MINV 280
C                                                                             MINV 290
C     ..................................................................      MINV 300
C                                                                             MINV 310
      SUBROUTINE MINV(A,N,D,L,M)                                              MINV 320
      DIMENSION A(1), L(1), M(1)                                              MINV 330
                                                                              MINV 340
```

116

```
C                                                               MINV 350
C     ..........................................................MINV 360
C                                                               MINV 370
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE MINV 380
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION     MINV 390
C        STATEMENT WHICH FOLLOWS.                                      MINV 400
C                                                                      MINV 420
C        DOUBLE PRECISION A,D,BIGA,HOLD                                MINV 410
C                                                                      MINV 430
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS   MINV 440
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS     MINV 450
C        ROUTINE.                                                      MINV 460
C                                                                      MINV 470
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO     MINV 480
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  ABS IN STATEMENT MINV 490
C        10 MUST BE CHANGED TO DABS.                                   MINV 500
C                                                                      MINV 510
C     ..........................................................MINV 520
C                                                               MINV 530
C        SEARCH FOR LARGEST ELEMENT                             MINV 540
C                                                               MINV 550
      D=1.0                                                     MINV 560
      NK=-N                                                     MINV 570
      DO 80 K=1,N                                               MINV 580
      NK=NK+N                                                   MINV 590
      L(K)=K                                                    MINV 600
      M(K)=K                                                    MINV 610
      KK=NK+K                                                   MINV 620
      BIGA=A(KK)                                                MINV 630
      DO 20 J=K,N                                               MINV 640
      IZ=N*(J-1)                                                MINV0650
      DO 20 I=K,N                                               MINV 660
      IJ=IZ+I                                                   MINV 670
   10 IF( ABS(BIGA) - ABS(A(IJ))) 15,20,20                      MINV 680
```

117

```
15 BIGA=A(IJ)                                              MINV 690
   L(K)=I                                                  MINV 700
   M(K)=J                                                  MINV 710
20 CONTINUE                                                MINV 720
                                                           MINV 730
C                                                          MINV 740
C      INTERCHANGE ROWS                                    MINV 750
C                                                          MINV 760
   J=L(K)                                                  MINV 770
   IF(J-K(  35,35,25                                       MINV 780
25 KI=K-N                                                  MINV 790
   DO 30 I=1,N                                             MINV 800
   KI=KI+N                                                 MINV 810
   HOLD=-A(KI)                                             MINV 820
   JI=KI-K+J                                               MINV 830
   A(KI)=A(JI)                                             MINV 840
30 A(JI) =HOLD                                             MINV 850
C                                                          MINV 860
C      INTERCHANGE COLUMNS                                 MINV 870
C                                                          MINV 880
35 I=M(K)                                                  MINV 890
   IF(I-K)  45,45,38                                       MINV 900
38 JP=N*(I-1)                                              MINV 910
   DO 40 J=1,N                                             MINV 920
   JK=NK+J                                                 MINV 930
   JI=JP+J                                                 MINV 940
   HOLD=-A(JK)                                             MINV 950
   A(JK) =A(JI)                                            MINV 960
40 A(JI) =HOLD                                             MINV 970
C                                                          MINV 980
C      DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS    MINV 990
C      CONTAINED IN BIGA)                                  MINV1000
45 IF(BIGA) 48,46,48                                       MINV1010
46 D=0.0                                                   MINV1020
```

118

```
      RETURN
   48 DO 55 I=1,N                                              MINV1030
      IF(I-K) 50,55,50                                         MINV1040
   50 IK=NK+I                                                  MINV1050
      A(IK)=A* IK)/(-BIGA)                                     MINV1060
   55 CONTINUE                                                 MINV1070
C                                                              MINV1080
C     REDUCE MATRIX                                            MINV1090
C                                                              MINV1100
      DO 65 I=1,N                                              MINV1110
      IK=NK+I                                                  MINV1120
      HOLD=A(IK)                                               MINV1130
      IJ=I-N                                                   MINV1140
      DO 65 J=1,N                                              MINV1150
      IJ=IJ+N                                                  MINV1160
      IF(I-K)  60,65,60                                        MINV1170
   60 IF(J-K)  62,65,62                                        MINV1180
   62 KJ=IJ-I+K                                                MINV1190
      A(IJ)=HOLD*A(KJ)+A(IJ)                                   MINV1200
   65 CONTINUE                                                 MINV1210
C                                                              MINV1220
C     DIVIDE ROW BY PIVOT                                      MINV1230
C                                                              MINV1240
      KJ=K-N                                                   MINV1250
      DO 75 J=1,N                                              MINV1260
      KJ=KJ+N                                                  MINV1270
      IF(J-K)  70,75,70                                        MINV1280
   70 A(KJ)=A(KJ)/BIGA                                         MINV1290
   75 CONTINUE                                                 MINV1300
C                                                              MINV1310
C     PRODUCT OF PIVOTS                                        MINV1320
C                                                              MINV1330
      D=D*BIGA                                                 MINV1340
C                                                              MINV1350
                                                               MINV1360
```

```
C     REPLACE PIVOT BY RECIPROCAL                    MINV1370
C                                                    MINV1380
      A(KK)=1.0/PIGA                                 MINV1390
   80 CONTINUE                                       MINV1400
C                                                    MINV1410
C     FINAL ROW AND COLUMN INTERCHANGE               MINV1420
C                                                    MINV1430
      K=N                                            MINV1440
  100 K=(K-1)                                        MINV1450
      IF(K) 150,150,105                              MINV1460
  105 I=L(K)                                         MINV1470
      IF(I-K) 120,120,108                            MINV1480
  108 JQ=N*(K-1)                                     MINV1490
      JR=N*(I-1)                                     MINV1500
      DO 110 J=1,N                                   MINV1510
      JK=JQ+J                                        MINV1520
      HOLD=A(JK)                                     MINV1530
      JI=JR+J                                        MINV1540
      A(JK)=-A(JI)                                   MINV1550
  110 A(JI)  =HOLD                                   MINV1560
  120 J=M(K)                                         MINV1570
      IF(J-K) 100,100,125                            MINV1580
  125 KI=K-N                                         MINV1590
      DO 130 I=1,N                                   MINV1600
      KI=KI+N                                        MINV1610
      HOLD=A(KI)                                     MINV1620
      JI=KI-K+J                                      MINV1630
      A(KI)=-A(JI)                                   MINV1640
  130 A(JI)  +HOLD                                   MINV1650
      GO TO 100                                      MINV1660
  150 RETURN                                         MINV1670
      END                                            MINV1680
```

120

APPENDIX II

EXTENDED KALMAN FILTER NOMENCLATURE LIST
AND PROGRAM LISTING

Nomenclature List

PROGRAM VARIABLE	MATH SYMBOL	DEFINITION
H	Δt	Numerical integration step size (sec)
TZERO	t_o	Initial time for numerical integration (sec)
TMAX	t_{max}	Cutoff time for numerical integration (sec)
ITO		Frequency of numerical integration output
N	N	Number of first order equations to be integrated
M	M	Number of data points
XZ(I)		Initial conditions for state variable and variance estimates
Q	q	System noise variance (rad^2/sec^4)
R	r	Measurement error variance (rad^2)
Z(I)	z	Experimental data values of α
X(1)	$\hat{x}_1$	Estimated value of α (rad)

PROGRAM VARIABLE	MATH SYMBOL	DEFINITION
X(2)	$\hat{X}_2$	Estimated value of $\dot{\alpha}$ (rad/sec)
X(3)	$\hat{X}_3$	Estimated value of C_3
X(4)	$\hat{X}_4$	Estimated value of C_4
X(5)	$\hat{X}_5$	Estimated value of C_5
X(6)	$\hat{X}_6$	Estimated value of C_6
X(7)	P_{11}	Variance of $\hat{X}_1$
X(8)	P_{12}	Covariance of $\hat{X}_1$ and $\hat{X}_2$
X(9)	P_{13}	Covariance of $\hat{X}_1$ and $\hat{X}_3$
X(10)	P_{14}	Covariance of $\hat{X}_1$ and $\hat{X}_4$
X(11)	P_{15}	Covariance of $\hat{X}_1$ and $\hat{X}_5$
X(12)	P_{16}	Covariance of $\hat{X}_1$ and $\hat{X}_6$
X(13)	P_{22}	Variance of $\hat{X}_2$
X(14)	P_{23}	Covariance of $\hat{X}_2$ and $\hat{X}_3$
X(15)	P_{24}	Covariance of $\hat{X}_2$ and $\hat{X}_4$
X(16)	P_{25}	Covariance of $\hat{X}_2$ and $\hat{X}_5$
X(17)	P_{26}	Covariance of $\hat{X}_2$ and $\hat{X}_6$
X(18)	P_{33}	Variance of $\hat{X}_3$
X(19)	P_{34}	Covariance of $\hat{X}_3$ and $\hat{X}_4$
X(20)	P_{35}	Covariance of $\hat{X}_3$ and $\hat{X}_5$

PROGRAM VARIABLE	MATH SYMBOL	DEFINITION
X(21)	P_{36}	Covariance of $\hat{X}_3$ and $\hat{X}_6$
X(22)	P_{44}	Variance of $\hat{X}_4$
X(23)	P_{45}	Covariance of $\hat{X}_4$ and $\hat{X}_5$
X(24)	P_{46}	Covariance of $\hat{X}_4$ and $\hat{X}_6$
X(25)	P_{55}	Variance of $\hat{X}_5$
X(26)	P_{56}	Covariance of $\hat{X}_5$ and $\hat{X}_6$
X(27)	P_{66}	Variance of $\hat{X}_6$
A(I)	$\dot{X}$	Derivative of X with respect to time

Categories of Input Data

Numerical integration constants.--These constants are
the numerical integration step size, frequency of output,
time at which the integration is to stop, initial time, the
number of first order equations being integrated, and the
number of data points to be read.

Initial conditions.--These data are the initial estimates
of the state variable components, state variable variances, and
covariances.

Noise variance estimates.--These two entries are
the estimates of the measurement error variance and system
noise variance.

Pitch angle data.--These data are the experimental
values of the pitch angle from which the filter estimates
the states and parameters. The time between data points
is not input but is assumed to be the same as the numerical
integration step size input on the first data card.

```
C  EXTENDED KALMAN FILTER WITH AUGMENTED STATE VECTOR NON LINEAR
C
      DIMENSION X(162),XZ(27),Z(500),AC(500)
      COMMON N,T,X
      COMMON Q,Z,J,JJITO,KI,R,ITD,JJJ,AC
      EXTERNAL XDOT,OUT
      READ (5,501,END=999) H,TZERO,TMAX,ITO,N,M,ITD,NP
501   FORMAT (3F10.4,5I10)
      READ (5,502) (XZ(I),I=1,N)
502   FORMAT (5E15.5)
      READ (5,503) Q,R
503   FORMAT (2E15.5)
      READ (5,504) (Z(I),I=1,M)
504   FORMAT (10F8.5)
      WRITE (6,601) H,TZERO,TMAX,ITO,N,M,ITD
6010  FORMAT ('1',5X,'INCREMENT=',F5.3,'SEC',5X,'TZERO=',F5.3,5X
     1'TMAX=',F5.3,5X,'ITO=',I3,5X,'N=',I3,5X,'M-',I3,5X,'ITD=',I3)
      WRITE (6,602) (I,XZ(I),I=1,N)
6020  FORMAT (1H0,5X,'XZ(',I2,')=',E15.5,10X,'XZ(',I2,')=',E15.5)
      WRITE (6,603) Q,R
603   FORMAT (1H0,5X,'Q=',E15.5,5X,'R=',E15.5)
      WRITE (6,604) (I,Z(I),I=1,M)
6040  FORMAT (1H0,5X,'Z(',I3,')=',F8.5,5X,'Z(',I3,')=',F8.5,5X,'Z(',I3,
     1')=',F8.5,5X,'Z(',I3,')=',F8.5,5X,'Z(',I3,')=',F8.5)
      JJJ=0
      J=0
      JJITO=ITO
      KI=-1
      CALL ADDUM (H,ITO,TZERO,TMAX,XZ,XDOT,OUT)
      IF(NP.EQ.0) GO TO 990
      WRITE(7,900) (AC(1),I=1,M)
900   FORMAT(10F8.5)
990   GO TO 001
999   STOP
      END
```

```
SUBROUTINE ADDUM(AITCH,ITO,TZERO,TMAX,X2,3,OUT)      ADDU 110
DIMENSION XZ(27),C(27,6),X(27,6)                     ADDU 120
COMMON N,T,X,                                        ADDU 130
H = ABS(AITCH)                                       ADDU 140
IT -ITO-1                                            ADDU 150
D=TZERO-TMAX                                         ADDU 160
IF (D)2,1,1                                          ADDU 170
1 H=-H                                               ADDU 180
2 HH=0.5*H                                           ADDU 190
D=H/24.0                                             ADDU 200
T=TZERO                                              ADDU 210
ISET=0                                               ADDU 220
DO     3  I=1,N                                      ADDU 230
X(I,1)=XZ(I)                                         ADDU 240
3 X(I,6)=XZ(I)                                       ADDU 250
CALL F(X(1,5),1)                                     ADDU 260
CALL F(X(1,2),0)                                     ADDU 270
IF(ITO)5,5,4                                         ADDU 280
4    CALL OUT(1)                                     ADDU 290
5 DO     6 K=1,N                                     ADDU 300
6    C(K,1) = X(K,5)*HH                              ADDU 310
     X(K,1) = X(K,1) + C(K,1)                        ADDU 320
T=T+HH                                               ADDU 330
CALL F(C(1,2),3)                                     ADDU 340
DO      7 K=1,N                                      ADDU 350
7    C(K,2) = HH*C(K,2)                              ADDU 360
     X(K,1) = X(K,6) + C(K,2)                        ADDU 370
CALL F(C(1,3),3)                                     ADDU 380
DO     8 K=1,N                                       ADDU 390
8    C(K,3) = H*C(K,3)                               ADDU 400
     X(K,1) = X(K,6) + C(K,3)                        ADDU 410
T=T+HH                                               ADDU 420
CALL F(C(1,4),1)
DO     9 K=1,N
```

126

```
       C(K,4) = C(K,4)*H                                                    ADDU 430
       X(K,1)=X(K,6)+(C(K,1)+2.0*C(K,2)+C(K,3)+0.5*C(K,4))*0.33333333       ADDU 440
       X(K,6) = X(K,1)                                                      ADDU 450
    9  ISET = ISET+1                                                        ADDU 460
       GO TO (10,12,17),ISET                                                ADDU 470
   10  CALL F(X(1,5),0)                                                     ADDU 480
       DO    11 K= 1,N                                                      ADDU 490
   11  X(K,3)=X(K,5)                                                        ADDU 500
       GO TO    18                                                          ADDU 510
   12  CALL F(X(1,5),01)                                                    ADDU 520
       DO    13 K=1,N                                                       ADDU 530
   13  X(K,4)=X(K,5)                                                        ADDU 540
       GO TO    18                                                          ADDU 550
   14  T=T+H                                                                ADDU 560
       DO    15 K=1,N                                                       ADDU 570
       X(K,1)=X(K,6)+D*(55.0*X(K,5)-59.0*X(K,4)+37.0*X(K,3)-9.0*X(K,2))     ADDU 580
       X(K,2)=X(K,3)                                                        ADDU 590
       X(K,3)=X(K,4)                                                        ADDU 600
       X(K,4)=X(K,5)                                                        ADDU 610
   15  X(K,5)=X(K,6)                                                        ADDU 620
       CALL F(X(1,5),1)
       DO      16 K = 1,N
       X(K,6)=X(K,6)+D*(9.0*X(K,5)+19.0*X(K,4)-5.0*X(K,3)+X(K,2))           ADDU 640
   16  X(K,1)=X(K,6)
       ISET = 3
   17  CALL F(X(1,5),0)                                                     ADDU 670
   18  IF (IT(20,19,20
   19  IT = ITO                                                             ADDU 690
       CALL OUT(())                                                         ADDU 700
   20  IT = IT - 1                                                          ADDU 710
       IF(ABS(TMAX-T)-ABS(HH))21,21,22
   21  RETURN                                                               ADDU 730
       GO TO  (5,5,14),ISET
   22  END                                                                  ADDU 750
```

127

```
SUBROUTINE ADER (N,T,K)
DIMENSION X(162),A(27),Z(500)
COMMON N,T,X
COMMON Q,Z,J,JJITO,KI,R,ITD,JJJ,AC
II=0
IF(J.GT.ITD) II=1
IF(K.EQ.3) GO TO 81
KI=KI+K
IF(KI.NE.0) GO TO 70
IF(K.EQ.0) GO TO 70
J=J+1
70 ADIF=(Z(J)-(Z(J)-Z(J+1))*(KI/JJITO))-X(1)
IF(KI+1).LT.JJITO) GO TO 81
IF(K.EQ.1) GO TO 81
KI=-1
81 E=2*X(1)*X(2)*X(6)+X(3)+3*X(1)**2*X(5)
B=X(4)+X(6)*X(1)**2
C=X(1)**3
D=X(1)**2*X(2)
A(1)=X(2)+(X(7)/R)*ADIF
A(2)=-(X(4)+X(6)*X(1)**2)*X(2)-(X(3)+X(5)*X(1)**2)*
X(1)+(X(8)/R)*ADIF
A(3)=(X(9)/R)*ADIF
A(4)=(X(10)/R)*ADIF
A(5)=(X(11)/R*ADIF
A(6)=(X(12)/R)*ADIF
A(7)=2*X(8)-X(7)**2/R
A(8)=X(13)-E*X(7)-B*X(8)-X(1)*X(9)-X(2)*X(10)-C*X(11)-
D*X(12)-X(7)*X(8)/R
A(9)=X(14)-X(7)*X(9)/R
A(10)=X(15)-X(7)*X(10)/R
A(11)=X(16)-X(7)*X(11)/R
A(12)=X(17)-X(7)*X(12)/R
A(13)=Q*II-2*(E*X(8)+B*X(13)+X(1)*X(14)+X(2)*X(15)+C*X(16)+
```

```
     D*X(17))-X(8)**2/R
     A(14)=-E*X(9)-B*X(14)-X(1)*X(18)-X(2)*X(19)-C*X(20)-
     D*X(21)-X(8)*X(9)/R
     A(15)=-E*X(10)-B*X(15)-X(1)*X(19)-X(2)*X(22)-C*X(23)-
     D*X(24)-X(8)*X(10)/R
     A(16)=-E*X(11)-B*X(16)-X(1)*X(20)-X(2)*X(23)-C*X(25)-
     D*X(26)-X(8)*X(11)/R
     A(17)=-E*X(12)-B*X(17)-X(1)*X(21)-X(2)*X(24)-C*X(26)-
     D*X(27)-X(8)*X(12)/R
     A(18)=-X(9)**2/R
     A(19)=-X(9)*X(10)/R
     A(20)=-X(9)*X(11)/R
     A(21)=-X(9)*X(12)/R
     A(22)=-X(10)**2/R
     A(23)=-X(10)*X(11)/R
     A(24)=-X(10)*X(12)/R
     A(25)=-X(11)**2/R
     A(26)=-X(11)*X(12)/R
     A(27)=-X(12)**2/R
     RETURN
     END
```

```fortran
      SUBROUTINE OUT (F)
      DIMENSION X(162),Z(500),AC(500),XPE(4)
      COMMON N,T,X
      COMMON Q,Z,J,JJITO,KI,R,ITD,JJJ,AC
      WRITE (6,608) T,(X(I),I=1,N)
 6080 FORMAT (1HO,5X,'T=',F6.3,11X,'X1=',E11.4,5X,'X2=',
     1E11.4,5X,'X3=',E11.4,5X,'X4=',E11.4,/6X,'X5=',E11.4,
     25X,'X6=',E11.4,5X,'Fil=',E11.4,4X,'P12=',E11.4,4X,
     3'P13=',E11.4,/6X,'P14=',E11.4,4X,'P15=',E11.4,4X,
     4'P16=',E11.4,4X,'P22=',E11.4,4X,'P23-',E11.4,/6X,'P24=',
     5E11.4,4X,'P25=',E11.4,4X,'P26=',E11.4,4X,'P33=',E11.4,
     64X,'P34=',E11.4,/6X,'P35=',E11.4,4X,'P36=',E11.4,4X,
     7'P44=',E11.4,4X,'P45=',E11.4,4X,"P46=',E11.4,/6X,
     8'P55=',E11.4,4X,'P56=',E11.4,4X,'P66=',E11.4)
      XPE(1) = (X(3)-160.)/1.6
      XPE(2) = (X(4)  -1.6)/0.016
      XPE(3) = (X(5)  - 1960.)/19.6
      XPE(4) = (X(6)  - 4.35)/0.0435
      WRITE(6,500) (XPE(I),I=1,4)
  500 FORMAT(6X,'PEX3=',E11.4,3X,'PEX4=',E11.4,3X,'PEX5=',E11.4,3X,'PEX6
     1=',E11.4)
      JJJ=JJJ+1
      AC(JJJ)=X(1)
      RETURN
      END
```

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13. ABSTRACT

An analysis of numerical methods for extracting aerodynamic coefficients from dynamic test data has been conducted. The emphasis of the analysis is on the effects that random measurement errors in the data and random disturbances in the system have on the accuracy with which the coefficients for linear and nonlinear systems can be determined. Both deterministic and stochastic methods for extracting the coefficients and determining their uncertainties are considered.

The deterministic technique considered, due to Chapman and Kirk, provides excellent estimates of both linear and nonlinear static pitching moment coefficients for the range of measurement efforts and system noise considered. Somewhat less accurate estimates of pitch damping coefficients are obtained.

The stochastic approach considered demonstrates the feasibility of using an extended Kalman filter, with a parameter augmented state vector, for determining the values of the aerodynamic coefficients and their uncertainties from noisy dynamic test data.

Parameter estimates obtained from the extended filter compare favorably with previously obtained results using deterministic techniques. Estimates of the parameter uncertainties provided by the filter are generally superior to those obtained with deterministic techniques particularly when system noise has corrupted the data.

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